# Applied Mechanics Reviews

*A Critical Review of the World Literature in Applied Mechanics*

A. W. Wundheiler, *Editor*

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GENERAL



MECHANICS OF SOLIDS



MECHANICS OF FLUIDS



HEAT



MISCELLANEOUS



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# Applied Mechanics Reviews

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**RUSSIAN TRANSLITERATION:** Excepting traditionally established spellings, the doubt-raising Russian letters will be substituted according to the following table representing a compromise between pronunciation and unique determination of the original spelling:

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August 1950 Vol. 3, No. 8

## Communications

**Concerning Rev. 3, 386, Feb. 1950: J. L. A. Cuperus and F. C. de Nie, Strengthening the road-bed of a railway.**

The dumping of the sand in the parallel ditches on both sides of the railway embankment was done only to prevent the sliding of the roadbed, not to eliminate the continual settlement. The original plan was to sustain the sand by steel-sheet piling to safeguard the pile foundations of adjacent premises from being damaged, but the execution of the test section of the work revealed that it was not necessary to use the planned sheet piling, so that considerable saving was obtained. F. C. de Nie, Holland

## Theoretical and Experimental Methods

(See also Revs. 1425, 1426, 1428, 1432, 1433, 1436, 1446, 1456, 1521, 1537, 1554, 1580)

1412. **Kent's mechanical engineers' handbook, 12th ed.: Design and production volume,** Colin Carmichael, editor, one volume, various pagings; **Power volume,** J. Kenneth Salisbury, editor, one volume, various pagings. New York: John Wiley and Sons, Inc.; London: Chapman and Hall, Ltd., 1950. \$8.50 each.

With minor exceptions, the design-and-production volume has been almost completely rewritten. New material, required by progress in many fields, has outweighed the obsolete material that has been discarded, resulting in a book more than three hundred pages longer than the previous edition. The power volume has been entirely rewritten, from cover to cover, excepting a few passages that have been revised by competent authorities and retained because of their value. From the prefaces

1413. **Fritz Oberhettinger and Wilhelm Magnus, Applications of elliptic functions in physics and technology (Anwendung der elliptischen Funktionen in Physik und Technik),** Berlin, Springer-Verlag, 1949, vii + 126 pp. \$3.90; bound, approx. \$4.60.

In practical applications of elliptic functions one needs the knowledge of a very large number of special formulas, and facility in handling them. The present book aims at introducing the reader to these formulas and illustrating various facets of the manipulative technique. The problems discussed have been selected with a view to giving the reader an all-round training in the mathematical technique, but they are grouped according to their nonmathematical subject matter.

Chapter I gives a brief summary of the theory of elliptic functions. Elliptic integrals, theta functions, elliptic functions of both Jacobi and Weierstrass, and transformations are treated. There is an excellent collection of useful formulas. It is natural to the plan of the whole work that no proofs should be given in this chapter, but references to books where readers could find proofs and additional formulas would have been useful. Apparently, the authors did not wish to quote their well-known book [*Formeln und Sätze für die speziellen Funktionen der mathematischen Physik*, 2nd ed., Berlin, Springer-Verlag, 1948; Rev. 2, **963**] where the reader will find all that is required.

Chapter II is devoted to conformal representation and the closely related theory of Green's functions in two-dimensional potential theory. Explicit results are given for the rectangle and for the ellipse, and for the regions inside or outside a regular polygon, in particular for the regions inside a square or equilateral triangle. In chapter III the authors show typical applications to two-dimensional electrostatics.

Chapter IV contains applications to problems of hydro- and aerodynamics. A brief discussion of vortices, single, or a regular pattern between two plane walls or in a channel of rectangular cross section, is followed by the theory of an infinitely long wing in a wind tunnel or rectangular cross section, with an appendix on tunnels of elliptic cross section. Chapter V is headed "miscellaneous examples," and it discusses the pendulum, the potential of a charged ellipsoid, the buckling of a column, and the Tchebysheff approximation.

The book is very well written, the exposition is lucid, the proofs are carefully arranged with just sufficient detail to enable the reader to follow every step, and yet not so detailed as to make him lose patience. There are references to the literature in every chapter (except the first one). At the end of the book the authors have provided 14 pages of numerical tables of complete and incomplete elliptic integrals which will enable the reader to perform numerical computations to a modest degree of accuracy without having to take recourse to more elaborate books or tables. No reference is given in the book as to the origin of these tables; according to information received from the authors they are extracted from Legendre's tables except for the table of the ratios of real and imaginary periods for which the source is Hayashi. A. Erdélyi, USA

1414. **Josef Lense, Expansion in series in mathematical physics (Reihenentwicklungen in der mathematischen Physik),** 2nd ed., Berlin: Walter de Gruyter and Co., 1947, viii + 226 pp. DM18 (approx. \$4.50).

The principal feature of this book [the first edition of which appeared in 1933] is a sound and rigorous presentation at a comparatively elementary level. From the point of view of the beginner, the almost complete absence of the phrase "it is easy to see" is particularly gratifying. All estimates needed in connection with series expansions, deformation of contours, etc., are carried out in detail: only routine algebra is left to the reader. An advanced student may find the going rather slow at times; but a beginner will be grateful for a moderate pace.

One might feel that the choice of the title is not very fortunate. This is not a book on the expansions occurring in mathematical physics, but rather a treatise on the properties of the special functions which enter into the expansions.

After a brief introduction on polynomial approximation and Fourier series, chapter I deals with asymptotic series, Bernoulli polynomials, the Euler-Maclaurin expansion, and approximate evaluation of definite integrals. Chapter II is on the gamma

function. The asymptotic behavior of the gamma function is investigated by means of the Euler-Maclaurin expansion. In addition to the more usual topics, the conformal mapping defined by the gamma function is also discussed. Chapter III is an exposition of the more elementary properties of orthogonal systems of functions. The process of orthogonalization, Bessel's inequality, completeness, convergence in mean, Laguerre, Hermite, and Tchebysheff polynomials are the topics discussed in this chapter. Chapter IV, on Bessel functions, is the longest in the book. The solutions of the differential equations are obtained as definite integrals of the Laplace type, and then evaluated in infinite series. Recurrence relations, more general integral representations, and Hankel's asymptotic expansions follow. Then come Sommerfeld's integrals, Airy integrals, and Debye's asymptotic expansions obtained by the method of steepest descent. Definite integrals containing Bessel functions are investigated, and the conformal mapping defined by Bessel functions is briefly described.

The second largest chapter of the book is chapter V on Legendre functions. These are shown to arise when Laplace's partial differential equation is separated in spherical polar coordinates. Legendre polynomials are discussed first. Their generating function, the Fourier series representing them, recurrence relations, integral representations, zeros, asymptotic representations are obtained. Then the author turns to associated Legendre functions of integer degree and order. In addition to the topics already mentioned, we have the orthogonal property, the expansion of $x^n$ in a series of Legendre polynomials, the expansion of a plane wave in spherical waves, and the application of Legendre polynomials to mechanical quadrature. Legendre functions of the second kind are given slightly less space. Lastly, spherical surface harmonics, their orthogonal property, their addition theorem, and their application to the solution of boundary value problems of Laplace's equation are discussed. Chapter VI is devoted to ellipsoidal harmonics. Laplace's equation is separated in ellipsoidal coordinates, and the various types of Lamé polynomials are established. Relations of ellipsoidal surface harmonics with spherical surface harmonics, and the degenerate case of confocal quadrics of revolution are discussed briefly, but there is no adequate treatment of spheroidal harmonics. Theorems of the zeros of Lamé polynomials, the orthogonal property, and the application of Lamé polynomials to boundary-value problems of Laplace's equation close this final chapter.

The book is an excellent introduction to the special functions of mathematical physics. It is not only explicit, but also very precise. Historical remarks and references to the originators of particularly neat and attractive proofs will put the whole subject in perspective for the student, and will teach him to value elegance as much as thoroughness in exposition and precision in the statement of results. A. Erdélyi, USA

**1415. E. Bodewig, On types of convergence and on the behavior of approximations in the neighborhood of a multiple root of an equation** (in German, with Russian summary), Z. angew. Math. Mech. 29, 44–51 (1949); (in English), Quart. appl. Math. 7, 325–333 (1949).

This paper is concerned with the approximation of the roots of an equation. A measure of the rapidity of the convergence of a sequence is established and "the degree of convergence" is defined. It is thereby possible to close two "gaps" in the theory of approximation methods, namely, the comparison of the efficiency of various methods, and the dependence of the convergence on the multiplicity of the roots. It is proved that the convergence of many methods depends on the multiplicity, for example, in Newton's method and a generalization thereof which is given here; in Laguerre's method; and in a general method which furnishes a convergence of degree $n$. E. Frank, USA

**1416. Rudolf Zurmühl, On the numerical solution of linear system of equations by the Banachiewicz method of matrices** (in German, with Russian summary), Z. angew. Math. Mech. 29, no. 3, 76–84 (1949).

The author describes the method of Banachiewicz [Bull. Int. Acad. Polon. Sci. Cl. Sci. Math. Nat. Ser. A. Sci. Math. 1938, 393–404 (1939)] for solving linear algebraic equations; it consists essentially in decomposing the matrix $A$ into the product of two triangular matrices with opposite directions: $A = LR$, where $R$ has elements 1 in the diagonal. [The reviewer wishes to emphasize that Banachiewicz's method results by applying the Gaussian method in a special way, namely by computing for every derived system only the first equation and the first column and writing them down, while the remaining coefficients of every other equation of the same derived system are not computed until they come into the first row or column of a derived system. This yields precisely the method, the scheme and the operations of Banachiewicz and of the author, but without seeking the decomposition $A = LR$, although it is yielded automatically by the process. This variant of Gauss' method has a disadvantage which is concealed by the form which Banachiewicz and the author give it by starting from $A = LR$, but which appears immediately if it is treated in the above manner. In fact, by choosing by this rigid scheme only the first equation of every derived system, it is possible that the leading coefficient of a derived system is small or even vanishes so that division by it is troublesome or even impossible and the procedure must be modified.]

*Courtesy of Mathematical Reviews* E. Bodewig, Holland

**1417. Günter Opitz, The convergence of Theodorsen's method of conformal mapping of circlelike domains** (in German), Arch. Math. 2, 110–116 (1950).

**1418. Ernst Lammel, On Theodorsen's numerical method of conformal mapping of simply-connected domains** (in German), Monatsh. Math. 53, 257–267 (1949).

The first paper shows that the Picard iteration method applied to the Theodorsen-Garrick equation results in a uniformly convergent sequence if $|\rho'(\psi)/\rho(\psi)| < 1$, where $\rho(\psi)$ is the polar equation of the boundary. The second paper gives similar theorems. Equivalent results are given in S. E. Warschawski, Quart. appl. Math. 3, 12–28 (1945). Ed.

**1419. Fritz Riegels, Formulas and tables for an elliptic integral appearing in the space potential theory** (in German), Arch. Math. 2, no. 2, 117–121 (1949/50).

Analysis and tables are given of the function

$$G_n(k^2) = (-1)^n \int_0^{\pi/2} (1 - k^2 \sin^2 \theta)^{-3/2} \cos 2n\theta \, d\theta$$

for $n = 0, 1, 2,$ and 3. Stephen H. Crandall, USA

**1420. Douglas Payne Adams, An index of nomograms,** New York, John Wiley & Sons, Inc.; London, Chapman and Hall, 1950, x + 174 pp. Cloth, 7.5 × 9.5 in., $4.

The author has compiled a list of approximately 1700 alignment diagrams published since 1923 in about 100 American periodicals. Index A contains key words for each of the diagrams with reference to index B which is arranged into 21 divisions corresponding to major fields of interest, varying from mathematics to food, and presents the periodical, month, year, volume, number, and page of each nomogram as well as the variables employed. A

future edition would be much improved by stating the ranges of the variables. Fr. A. Willers, Germany

1421. **K. Stange, On the rational reduction of discontinuous time-of-travel records** (in German), Ing.-Arch. 16, 383–402 (1948).

It is supposed that a sequence of points at equal intervals on a curve have been determined with a given precision, and that a smoothed value of the ordinate to the curve, as well as values of the first and second derivatives, are obtained at each point by fitting a polynomial by the method of least squares to a specified number of neighboring points. The extent of reduction of error in the ordinate and the derivatives effected through the smoothing and the amount of distortion resulting therefrom are discussed in relation to the degree of polynomial and the number of neighboring points used in the smoothing process.

*Courtesy of Mathematical Reviews* T. N. E. Greville, USA

1422. **H. Charpentier, Introduction to dimensional methods** (in French), Bull. Ass. tech. Marit. Aéro. 46, 137–170 (1947).

The author gives a synthetic and critical outline of modern dimensional methods. He reaches the conclusion that these methods should be considered only as a mathematical tool which may, very often and as an auxiliary to experience, facilitate conjecture. He particularly insists on the singleness of the theory in its three aspects: units, conjecturing, simplitude; the auxiliary nature of the method; the conventional nature of the choice of the fundamental quantities; the relativity of dimensional formulas; the dual character of units and similitude; the restrictions in similitude; the role of numerical factors; and the importance of similitude in experimental research.

M. Bricas, Greece

## Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 1431, 1434, 1437, 1459)

1423. **Wilhelm Blaschke, On the geometry of motion on a sphere** (in German), S.-B. Heidelberger Akad. Wiss. Math.-Nat. Kl. 1948, no. 2, 9 pp. (1948).

Using "moving frames," the author develops the formula for a (closed, i.e., initial state = final state) one-parameter motion of the plane and the sphere, and discusses the concepts of space pole, body pole, Steiner point, and Steiner vector. A relation is obtained between the total geodesic curvature of the path of an arbitrary point $x$, the Steiner vector, and the index of the body-pole curve with respect to $x$.

*Courtesy of Mathematical Reviews* H. Samelson, USA

1424. **K. Brokate, Calculation of the curve of the cardioid gearing with two rollers** (in German), Feinwerk Tech. 53, 49–53 (May 1949).

This is a study of the transformation of a uniform rotation into a uniform alternating rectilinear motion, with a view to the practical realization and with particular attention to the dead positions. A numerical example is worked out.

Gino Moretti, Argentina

1425. **K. Beyerle, The slip error of the disc-and-wheel integrator** (in German), Z. angew. Math. Mech. 29, p. 186 (1949).

The small friction load imposed by the counter-mechanism in a disk-and-wheel integrator produces an error by enabling the integrator to follow a line slightly inclined to a radius of the disk without registering the rotary component. In a test with a precision integrator measuring the area of a narrow right triangle lying near a radius, this error amounted to about 3%.

M. Goldberg, USA

1426. **E. W. Kammer and Sherwood Holt, Jr., A bonded-wire strain-gage type accelerometer,** Proc. Soc. exp. Stress Anal. 6, no. 2, 53–60 (1948).

The authors describe the construction and calibration of the title instrument, having tubular load-sensitive members, stressed in tension. The authors state that its useful range is from 20 to 5000 g, its maximum frequency is 5000 cycles per sec, and its accuracy is well within 10%.

Alexander Yorgiadis, USA

1427. **Arnold Sommerfeld, Mechanics of deformable bodies (Lectures on theoretical physics, vol. 2),** translated by G. Kuerti; New York, Academic Press, 1950, viii + 396 pp. Cloth, 5.3 × 7.8 in., $6.60.

In this translation of the second volume of Sommerfeld's *Vorlesungen über theoretische Physik*, the text of the second edition (1947) has been followed [Rev. 3, **1023**]. Disregarding a few nontechnical remarks that hold interest only for German readers, nothing has been omitted, and no changes have been made apart from necessary adaptations of the notation. Several brief footnotes have been inserted, however, to bridge over certain differences, mostly of a terminological nature. Topical references to the first volume have been supplemented by corresponding references to well-known American and British texts with the aim of making this volume independent from the preceding one, and a new index has been prepared. Translator's note

1428. **Ugo Dal Buono, On the solution of a classical problem** (in Italian), Boll. Un. Mat. Ital. 3, no. 3, 248–250 (Dec. 1948).

The author presents a simple solution of the following classical problem (see Tait and Steele, *Dynamics*): A homogeneous chain of constant total length hangs on a small pulley (without friction) so that one of its ends is free while on the other side the chain lies partly on a fixed horizontal plane. Thus the mass of the moving part of the chain varies during the motion.

Z. Horák, Czechoslovakia

1429. **M. Sh. Aminov, On the stability of certain mechanical systems** (in Russian), Prikl. Mat. Mekh. 12, no. 5, 643–646 (Sept.-Oct. 1948).

For a system of kinetic energy $T$, potential energy $V$ and total energy $h$, certain sufficient stability conditions are expressed in terms of the coefficients of the quadratic form $(h - V)Tdt^2$.

Ed.

## Gyroscopics, Governors, Servos

1430. **Cataldo Agostinelli, On the motion of a rigid, heavy, asymmetric body with the mass center lying to the axis of one of the cyclic planes of the ellipsoid of inertia** (in Italian), Atti semin. mat. fis. Univ. Modena 3, 248–260 (1948–1949).

G. Grioli [Ann. Mat. pura appl. 26 (1947); Rev. 2, **156**] discussed the existence of possible regular precessions in the motions of a heavy asymmetric body with a fixed point and found that regular precession occurs only when the body's center of gravity is situated on the axis of the cyclic planes of the ellipsoid of inertia referred to the fixed point.

In the present paper the possibility of other kinds of motions of the body, the center of gravity of which satisfies the above-mentioned conditions, is examined. The author starts from

Euler's equations for the general motion of a heavy body, and establishes the equation for this special case. The integration of these equations leads to the conclusion that only motions with regular precession are possible. Aurel A. Beleş, Rumania

**1431. G. Grioli, Stability questions concerning regular precessions of an asymmetric heavy rigid body** (in Italian), Ann. Scu. norm. sup. Pisa 1, 43–74 (1949).

In two foregoing papers [see preceding review and Rev. 2, **156**] the author found that in the motion of a heavy solid about a fixed point there exist (under certain conditions) $\infty^2$ regular precessions. He investigates the stability of these special types of motion. He used for this the method of the characteristic exponents established by H. Poincaré, which for a system of linear differential equations with periodical coefficients is however much more complicated than for constant coefficients. He shows that four of the six characteristic exponents are always zero, but in general two are not zero. By means of a new theorem of analysis he succeeded in determining the nature of the integral in the individual cases, and in showing that stability holds only in the case that the initial disturbances do not exceed a certain amount, and do not change the energy of the system and the moment of momentum about the vertical. Th. Pöschl, Germany

**1432. Herbert Bilharz, Stability criterion in balancing theory** (in German), Math. Nachr. 2, 314–316 (1949).

The Laplace transform is used to extend the criterion for stability of a linear differential equation with constant coefficients to the case of a functional-differential equation derived by inserting a lag in some of the terms.

*Courtesy of Mathematical Reviews* P. Franklin, USA

## Vibrations, Balancing

**1433. N. F. Barber, A simple frequency analyser which measures phase,** J. sci. Instrum. 26, 185–188 (June 1949).

The signal to be analyzed is made to vary in time the intensity of illumination of a screen consisting of a pattern of diverging transparencies. The shadow cast by this screen falls upon a moving belt of photographic paper. Upon development this paper shows a number of fringe patterns whose various spacings indicate the various periodicities in their correct relative phases. The frequency range and degree of resolution are easily altered. The strength of the various periodicities affects only the intensity of the fringes and so cannot be assessed accurately.

Author's summary

**1434. Max Weber, Contribution to the measurement of vibrations** (in German), Helv. phys. Acta 22, 425–456 (1949).

After a lengthy general discussion of vibration meters, the construction and calibration of an accelerometer utilizing a piezoelectric crystal beam as the moving part, together with an electronic amplifier and filter are described. S. H. Crandall, USA

**1435. A. H. J. M. Tomey, A new method of analysis of graphs produced by damped sinusoidal vibrations,** Pont. Acad. Sci. Comment. 11, 273–325 (1947).

**1436. Alfred Walz, A balancelike device for harmonic analysis** (in German), Z. angew. Math. Mech. 29, no. 1/2, 42–44 (1949) = Arch. Math. 1, 383–392 (1949).

A device for harmonic analysis is described which utilizes the Bessel summation formulas for numerical harmonic analysis by providing for introducing the $2n$ measured ordinates of the curve to be analyzed as the positions of unit weights on $2n$ levers which are capable of rotation around a common horizontal axis. For each Fourier coefficient obtainable there is a curved template with stairstep profile by means of which the levers can quickly be rotated close to the appropriate angles with respect to the vertical, and then by turning a long bolt of oval cross section through 90 deg they are locked accurately in correct relative position. The static moment about the axis then provides evaluation of the Bessel formula. Pictures of a preliminary model for 24 ordinates are shown. The moment principle has previously been suggested by various writers; the present design is advanced as a practical embodiment which at the same time avoids some of the disadvantages of planimeter-type analyzers. The author believes greater speed of operation and comparable accuracy are obtainable with such machines of moderate size if constructed with precision. In addition they can be used for point-by-point harmonic synthesis.

*Courtesy of Mathematical Reviews* R. Church, USA

**1437. N. V. Butenin, On the theory of forced oscillations in a nonlinear mechanical system with two degrees of freedom** (in Russian), Prikl. Mat. Mekh. 13, 337–348 (1949).

The author discusses a mechanical system with two degrees of freedom, close to a linear conservative system, and undergoing a forced harmonic oscillation whose period is far from the two fundamental periods of the linear conservative system. The basic equations assume the form

$$\ddot{x} - k_1\dot{y} - n_1^2x = \mu f(x,\dot{x},y,\dot{y}),$$

$$\ddot{y} + k_2\dot{x} - n_2^2y = \mu g(x,\dot{x},y,\dot{y}) + Q\sin t.$$

One looks for a solution of the form:

$$x = a\sin(\omega_1 t + \varphi_1) + b\sin(\omega_2 t + \varphi_2) + d\cos t,$$

$$y = (ak_1 + a_2)\cos(\omega_1 t + \varphi_1 - \gamma_1) + b(bk_2 + b_n)\cos(\omega_2 t + \varphi_2 - \gamma_2) + c\sin t.$$

For $a_2 = b_2 = \gamma_1 = \delta_2 = 0$, and $a,b,\ldots$ all constant, this is the solution for $\mu = 0$. For $\mu \neq 0$ but small, it is assumed that $c,d$ are the same as when $\mu = 0$, and $a,b,\varphi_1,\varphi_2$ are slowly varying parameters with $a_2,b_2,\gamma_1,\gamma_2$ corrective terms of order $\mu$. The approximate treatment of the system generalizes the well-known methods characteristic, for example, of the work of Kryloff-Bogolyuboff [see *Introduction to nonlinear mechanics* (1937); Ann. Math. Studies no. 11]. The treatment is applied to forced oscillations of a mechanical system (monorail car) with gyroscopic stabilizer. [See N. V. Butenin, same source, 10, no. 11 (1940).]

S. Lefschetz, USA

**1438. D. I. Sherman, On the theory of steady oscillations of a medium under given exterior forces on its boundary** (in Russian), Prikl. Mat. Mekh. 13, no. 5, 557–560 (1949).

The author considers the steady oscillation of an elastic medium which lies in a simply connected plane region $S$ bounded by a "sufficiently smooth" curve $L$. The exterior forces acting on the medium are given on $L$. The longitudinal and transverse potentials, $\phi(x, y)$ and $\psi(x, y)$, respectively, must satisfy the equations $a^2\Delta\phi + \gamma^2\phi = 0$, and $b^2\Delta\psi + \gamma^2\psi = 0$, where $a$ and $b$ are the velocities of propagation of the longitudinal and transverse waves respectively, $\Delta$ is the Laplacian, while $\gamma$ is the frequency of the oscillation. The problem consists in finding functions $\phi(x, y)$ and $\psi(x, y)$ satisfying the above equations in $S$ and also the imposed boundary conditions along the curve $L$. This problem is re-

duced to the solution of a pair of integral equations of the Fredholm type. The author states that the existence proof for the solution of this system for almost all values of $\gamma$ is analogous to that given in an earlier paper [Prikl. Mat. Mekh. 11, 259–266 (1947); Rev. 1, **395**].

*Courtesy of Mathematical Reviews* H. P. Thielman, USA

1439. **Dario Graffi, On the theory of free vibrations of an elastic hereditary system** (in Italian), R. C. Semin. Mat. Fis. Univ. Modena 3, 21 pp. (1948–49).

A treatment by Enrico Volterra [Rev. 1, **940**] of a simply supported beam with elastic after-effect is extended to the case where the after-effect function is the sum of a finite number $n$ of exponential terms. Robert E. Roberson, USA

1440. **P. Draminsky, Crankshaft damping,** Proc. Inst. mech. Engrs. 159, issue 46, 416–432 (1948).

The author attempts to give an explanation of the damping by torsional vibrations in motor crankshafts, and also to obtain formulas for precalculation of the damping in any given case. The paper describes experimental work with a single-cylinder engine driven by external power and excited to torsional vibrations by a spring-loaded cam disk. The paper also gives a method for the calculation of damped vibrations in elastic systems, and formulas are given for the damping in multicylinder engines with or without heavy flywheels, and the results are compared with the measured damping of oil engines in service. The investigations described in the paper show that the principal factors which influence damping are: bearing clearance, vibrations per revolution, arrangement of counterweights, and number of cylinders. In addition, the experimental work revealed certain unexpected results which may be of value in other fields. For a mild-steel shaft the hysteresis damping factor was found to be absolutely constant for torsional stresses up to the fatigue limit; at higher stresses the damping rose sharply. Therefore hysteresis measurements could be useful in determining exact fatigue limits. The author also found that short bearings have a greater load capacity than that indicated by the common hydrodynamic theory, because of the low oil temperature due to the increased oil flow through the bearings. A. R. Holm, Denmark

1441. **Walter Ramberg, Transient vibration in an airplane wing obtained by several methods,** J. Res. nat. Bur. Stands. 42, 437–447 (May 1949).

The author analyzes the flexural transients in a model airplane wing following a simulated landing impact. When the impact duration is of the order twice the fundamental flexural period, the method of Williams, which separates the static and dynamic response, is found to give a maximum bending moment of 107% of the measured value. Using the same three symmetric and three antisymmetric modes, the normal modes method of Biot and Bisplinghoff gives only 77% accuracy. A third method developed by Levy uses special coupled modes, and gives 105% accuracy with only three modes. In this example a simple rigid-body method yields 85% of the measured value.

None of these methods was found to be satisfactory for an impact duration of the order of one third of the fundamental period. Attempts to develop a traveling-wave method, analogous to that used by Saint Venant to solve the problem of longitudinal impact of an elastic bar, were made so far without success.

Incidentally, the author shows that a more general method of separating the static and vibrational response of an undamped system may be obtained immediately by writing Duhamel's integral in the form:

$$q(t) = \frac{Q(t)}{K} - \frac{1}{M\omega^2}\int_0^t \cos\omega(t-\tau)\dot{Q}(\tau)d\tau.$$

P. C. Dunne, England

1442. **C. W. Prohaska, Vertical vibrations of ships** (in French), Bull. Assoc. tech. Marit. Aéro. 46, 171–219 (1947).

The paper discusses the two-noded vibration of the ship as a free-free beam in bending in a vertical plane. It uses the old Schlick formula $\omega^2 = \text{const}\ I/L^3\Delta$, where $I$ is the moment of inertia of the principal cross section, $L$ the length, $\Delta$ the displacement. The constant depends on many factors such as entrained water, stiffness and weight distribution along the length, shear forces, and depth of water. For all of these factors numerical tables and graphs are given, obtained empirically and semi-analytically. With the aid of these tables the coefficient can be quickly determined, and the frequency thus found agrees within 4% with experimental frequencies found in 25 cases.

J. P. Den Hartog, USA

1443. **Charles Legreau, Dynamic balancing of rotating parts** (in French), Mach. Mod. 43: no. 475, 33–39 (Jan. 1949); no. 467, 29–32 (Feb. 1949); no. 477, 10–16 (Mar. 1949).

In a simple and comprehensive way the author presents the theory of dynamic balancing. The various apparatus and techniques in common use are described and explained.

R. G. Boiten, Holland

## Wave Motion, Impact

(**See also Revs. 1441, 1450, 1476, 1518, 1529, 1559, 1612**)

1444. **F. J. Meister, Measurement of mechanical shock waves** (in German), Arch. tech. Messen, no. 161, V172-2 (May 1949).

This paper deals with the characteristics of devices for correctly recording shock phenomena (pressures, motions, etc.). A shock phenomenon can be represented as an infinite number of periodic partial oscillations. These cover a range of frequencies which becomes greater as the build-up and die-down times of the shock decrease. To record such a phenomenon reliably, the recording mechanism must have constant magnification and phase displacement over the whole range of frequencies represented. The author discusses the properties of three types of devices for recording: (1) those with very high natural frequency and very low damping (piezoelectric, condenser and magnetostrictive gages); (2) those with very low natural frequency and considerable damping; (3) those using a series of contacts, each opening or closing at a predetermined level of the phenomenon being studied, giving records on film or otherwise. M. P. White, USA

1445. **J. P. Walsh and R. E. Blake, The equivalent static accelerations of shock motions,** Proc. Soc. exp. Stress Anal. 6, no. 2, 150–158 (1948).

The authors describe a "reed gage" for recording the maximum deflection of the masses at the end of several cantilever reeds when the case of the gage is subjected to the shock under investigation. They define the "equivalent static acceleration" of the shock acting on a given reed as the ratio of the recorded deflection of the mass at the end of that reed to the cantilever-beam deflection of the mass under its own weight. The equivalent static acceleration obviously depends on the frequency of the reed, as well as on the velocity of the shock motion as a function of time. The damaging effect of the shock motion on an elastic structure can be estimated from the "shock spectrum" which is a plot of the equivalent static acceleration for a large number of reeds of dif-

ferent frequencies, provided that each reed can be regarded as a linear, frictionless single-degree-of-freedom system, and provided that the reed gage itself does not modify the shock motion.

Shock spectra are presented for two Navy high-impact shock machines. These bring out differences in the shock at different portions of the anvil plate of one machine.

Walter Ramberg, USA

## Elasticity Theory

(See also Revs. 1438, 1439, 1465, 1602)

1446. **K. Hohenemser, Methods of approximate solution of eigenvalue problems in elastokinetics (Die Methoden zur angenäherten Lösung von Eigenwertproblemen in der Elastokinetik),** New York, Chelsea Publishing Company, 1949, 89 pp. Cloth, 5.5 × 8.7 in., $2.75.

This is a reprint of the well-known 1933 book giving a referenced survey of several iteration processes (Stodola, Rayleigh, van den Dungen, Ritz, Galerkin, Trefftz), finite-difference methods, perturbation methods, and a number of specialized problems.

Ed.

1447. **Wolf Gross, Green's matrix for a problem of plane elasticity** (in Italian), Pubbl. Ist. appl. Calc. no. 255, 5 pp. (1949).

The author finds the displacements for a circular disk of elastic isotropic material in equilibrium under a given distribution of forces applied at the circumference. A general method for the solution of such problems has already been given by Muschelishvili [see e.g., Z. angew. Math. Mech. 13, 264–282 (1933)].

L. M. Milne-Thomson, England

1448. **F. Jung, The Culmann and Mohr circle** (in German), Öst. Ingen.-Arch. 1, no. 4/5, 408–410 (1947).

The paper presents a neat construction for deriving the circles of Culmann and Mohr showing their intimate relationship, together with some remarks about priority and credit for the introduction of graphical methods in the study of stress, strain, and inertia tensors.

Stephen H. Crandall, USA

1449. **Ubaldo Richard, On the problem of the clamped plate** (in Italian), Atti. Accad. Torino 83, 1, 21–27 (1949).

An elegant general solution is given of certain small displacement problems associated with an arbitrarily shaped thin flat plate clamped at its edges; the analysis applies to static problems of normal loading and dynamic problems of free or forced vibrations. It is shown, through the use of Green's functions, that the solution of a problem is obtained from the limit of a sequence of successive approximations which at any stage satisfies the equation of normal equilibrium, or of motion, but not all the boundary conditions.

H. G. Hopkins, England

1450. **J. D. Eshelby, Uniformly moving dislocations,** Proc. phys. Soc. Lond. Sect. A, 62, 307–314 (May 1949).

This paper is predominantly concerned with the calculation of the displacement vector in an isotropic elastic medium through which an edge dislocation is moving with uniform velocity $c$. On the basis of the theory of elasticity, it is shown that the velocity of shear waves $c_2$ fixes a limit beyond which the dislocation cannot be accelerated by applied forces. By an extension of a method due to Peierls [same source, 52, p. 34 (1940)], an estimate is then made of the effect of the atomic structure of the medium. Frank's results [same source, 62, p. 131 (1949)] for screw dislocations are reproduced, and the corresponding Peierls-Nabarro calculation given to show that the width of such a dislocation is proportional to $(1 - c^2/c_2^2)^{1/2}$. The elastic-wave equations are solved by means of the theory of Fourier transforms.

Ian N. Sneddon, Scotland

## Experimental Stress Analysis

(See also Revs. 1471, 1478)

1451. **S. K. Ghaswala, Elements of the theory of photoelasticity,** Civil Engng. Lond. 45: no. 524, 107–109 (Feb. 1950); no. 525, 167–169 (Mar. 1950); no. 526, 237–238 (Apr. 1950).

This is a general survey of the historical development, the materials and methods, and some applications of photoelasticity. A note on crazing of certain plastics is included.

Ed.

1452. **Robert R. Philippe and Frank M. Mellinger, A study of photoelastic models of foundations,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. 5, 58–62 (June 1948).

The authors describe developments in the technique of using gelatin models for photoelastic studies of stresses in foundations. The method as described is reported to be particularly useful when applied to cases involving complicated boundary and loading conditions.

A substantial section of the paper deals with model construction and calibration techniques. A minimum of theory is presented. The solution of the problem of evaluating the boundary stresses for a flood wall with a sloping base and vertical key is given to illustrate the techniques.

The following advantages and limitations are offered for the use of gelatin for photoelastic models of foundations: (a) It has greater optical sensitivity than any other known material from the point of view of double refraction, thereby permitting true scale loading. (b) It can be cast to any required shape, and is sensitive enough to allow the evaluation of stresses due to body forces. (c) Normal and shear stress determinations are within the requirements of engineering accuracy. (d) The assumption of elasticity for the foundation is made necessary by the method of test. (e) For the case illustrated, uplift forces on the wall section due to seepage are not simulated, although they can be determined separately and superimposed on the results. (f) Due to the rigid confinement of the glass sides of the tank, the gelatin foundation simulates a condition of true two-dimensional strain.

R. E. Fadum, USA

1453. **F. T. Barwell, "Frozen stress" in moulded polystryene,** Proc. int. rheolog. Congr. Holland, II: 232–238; III: 65–66 (1948).

Polystyrene impact-test specimens were compression molded at 145, 160, and 180 C. "Mottled" patterns were exhibited by those formed at the lower temperatures when viewed by transmitted polarized light. Specimens formed at the lower temperatures and heated for 24 hr at 120 C showed considerable surface roughening and general distortion, while only a slight bending distortion occurred for the specimens molded at the highest temperature. Rapid cooling of the mold resulted in stress as well as mottled patterns being formed. It is concluded that high molding temperatures should be used to prevent later specimen distortion, and that photoelastic inspection may be used as a manufacturing control.

Irwin Vigness, USA

1454. **Eugen Oszwald, The influence of a depth-dependent stress distribution on X-ray stress measurement** (in German), Z. Metallk. 39, no. 9, 279–288 (Sept. 1948).

A series of equations is derived which concern the effective

depth of penetration for X-rays in a stress field assumed to be changing with greater depth of penetration but exhibiting a stress gradient that remains constant. A discussion of the effects of an inhomogeneous stress field on different measuring techniques or use of various wave lengths is developed. Data obtained by the usual procedure of stress analysis, based on the assumption of the existence of a homogeneous stress field, are investigated and a relationship for re-evaluation of such data for conditions of an inhomogeneous stress field with stress changes in depth is established.

The order of magnitude of the influence of a depth-dependent stress distribution on the X-ray back-reflection measurements is investigated and substantiated by X-ray measurements of various workpieces. S. Weissman, USA

1455. **Bernard D. Haber, Experimental determination of aircraft loads,** Proc. Soc. exp. Stress Anal. 6, no. 2, 120–130 (1948).

Procedures are discussed for use in relating static-test loads to the actual loads in aircraft structures while in flight or when landing. The problem is complicated by the fact that flight loads must be determined indirectly from readings of electrical strain gages. This is ordinarily done by applying static loads to parts of an airplane structure in the laboratory and relating the strain-gage readings thus obtained to those obtained in flight. The author considers two examples: one is a landing gear to which the principle of superposition can be applied only approximately, and which on landing may have resonant dynamic conditions; the other is a semimonocoque boom for which the stresses are not proportional to the distance from the neutral axis.

This article seems unnecessarily hard to understand, and much of the blame should go to the one who edited it for publication. It may be mentioned that Equation (20e) is obviously wrong if strain is dimensionless, and that an "Appendix I" is mentioned, even though no appendices are in evidence.

W. C. Johnson, Jr., USA

1456. **F. W. Bubb, Use of network analyzers in the solution of thermal and stress problems,** Nucl. En. Power Aircr. Heat Transf. Lect. 1, 165–191 (Dec. 1948).

This is an expository paper on the simulation of heat flow in a flat body (or the plane distribution of stress) by current flow in a network of electrical resistances and capacitors. The conversion factors for both rectangular mesh and curvilinear mesh networks are derived. The methods may be extended to three dimensions. Approximations to various irregular shapes may be achieved. Boundary conditions correspond to constant (or controllable) values in coordinates. Michael Goldberg, USA

## Rods, Beams, Shafts, Springs, Cables, etc.

**(See also Rev. 1440)**

1457. **Ove Pettersson, Combined bending and torsion of simply supported beams of bisymmetrical cross section,** Trans. roy. Inst. Technol. Stockh. no. 29, 43 pp. (1949).

An analysis is presented for determining the stresses in a simply supported beam of bisymmetric cross section subjected to an eccentric oblique concentrated load at the center of the span. This is a difficult problem because of the nonlinearities which prevent the use of superposition. General equations are derived and then, by making reasonable simplifying assumptions, are reduced to a single fourth-order differential equation which governs $\varphi$, the angle of rotation. Expressions for the total normal stresses are developed in terms of the stresses in bending and torsion by adding a correction factor which is a function of the angle $\varphi$.

The differential equation governing $\varphi$ is solved for three cases of loading, namely, (1) eccentric load parallel to plane of maximum flexural rigidity, (2) oblique load passing through the centroid of the cross section, and (3) oblique load applied at any point. Solutions are obtained in series form from which numerical results are calculated and given in graphs. An example is given to illustrate the application of the method to a problem such as might arise in the design of a crane-bridge girder.

Dana Young, USA

1458. **J. A. Haringx, Elastic stability of flat spiral springs,** Appl. sci. Res. Sec. A A-2, no. 1, 9–30 (1949).

The author treats the problem of finding the angular deflection at which a flat spiral spring will become unstable. As a first step, a rough analysis is made, assuming that the spring has a large number of coils each of which may be considered as approximately circular in form. The results of this analysis indicate that for spiral springs having a large number of coils, where the ratio of long to short sides of the rectangular cross section is greater than about 10, the critical number of turns $\Delta N$ at which buckling occurs is equal to about 3 and is practically independent of the number of coils, the modulus of elasticity, or the dimensions of the spring. The analysis also indicates that in practice no more than, say, three turns may be obtained from such springs without instability. To investigate the effect of a small number of turns, an analysis is made of a fictitious spiral spring composed of a few coils of identical circular shape. Assuming that the ends of these coils may move radially without restraint, and that the elements of the coil are deflected slightly from the initial plane of the spiral, a differential equation for the deflection is set up. Solving this equation and using the boundary conditions of the problem, a system of three equations is obtained for the symmetric form of deflection and a similar system for the asymmetric case. The condition for buckling is obtained by setting the determinant of these equations equal to zero. The results indicate that the number of coils has an influence on the critical angle $\Delta N$ only for an initial number of turns less than about 8. A chart is given, based on the analysis, showing the effects of a smaller number of coils. The theoretical results appear to be in fair agreement with test data obtained on several springs. A. W. Wahl, USA

1459. **J. W. Campbell, The mechanical behavior of guy wires,** Canad. J. Res. Sec. F 27, 167–176 (Mar. 1949).

When a column is supported by a set of guy wires, each wire hangs in the form of a catenary, and when a lateral external pressure is exerted on the column, it must sway. In the paper the problem is treated by means of new catenary formulas which are especially convenient for the purpose. The method is illustrated by numerical examples, but the problem is treated only as statical, with no regard to the dynamical part of the problem (inertia, vibration) and no regard to what will happen if the guy wires are overloaded. A. H. Holm, Denmark

## Plates, Disks, Shells, Membranes

**(See also Revs. 1447, 1449, 1467, 1470)**

1460. **K. Ludwig, Bending of rectangular plates without the Bernoulli or other assumptions** (in German), Z. angew. Math. Mech. 29, 18–19 (Jan.-Feb. 1949).

By expressing the displacement components $\xi$, $\eta$, $\zeta$ as third-degree polynomials of $z$, the author shows that the coefficients $\xi_k$, $\eta_k$, $\zeta_k$ may be determined directly from the equations of motion and the boundary conditions, without using the Bernoulli or other assumptions. C. T. Wang, USA

**1461. F. Pongratz, The effective width of a reinforced concrete slab loaded with a single load and freely supported at two ends** (in German), Öst. Ingen.-Arch. 1, nos. 4/5, 351–371 (1947).

This paper deals with the problem of a reinforced-concrete slab, simply supported along two opposite edges, free along the other two, and loaded by a concentrated force. The results of a series of experiments are compared with formulas given by Nádái, based on the theory of isotropic plates. The main conclusions derived from this comparison are: (1) The loads producing the first cracks determined by experiments are fairly close to those given by the theory of isotropic plates, the value of Poisson's ratio having little influence on them. (2) The amount of transverse reinforcement does not affect the state of stress up to the load producing cracks, and does not affect the value of the yield load (the load producing yielding of the reinforcing steel). (3) The ratio of failure load to the yield load varies between 2 and 3, and is not always equal to approximately 1 as for a beam. Empirical formulas for the effective width, defined as the width along which a line load must be uniformly distributed in order to give the same moments as the concentrated load, are discussed. No references to investigations of the same problem are given.

G. Herrman, USA

**1462. Henri Pailloux, Stress determination in a membrane without stiffness** (in French), C. R. Acad. Sci. 228, 54–56 (Jan. 1949).

Using vector notation, the author obtains three partial differential equations of equilibrium satisfied by three quantities which specify the stress in a curved membrane without stiffness. [For equivalent tensor equations in the more general case of a shell, see J. L. Synge and W. Z. Chien, Theodore von Kármán Anniversary Volume, California Institute of Technology, Pasadena, Calif., 1941, pp. 103–120, in particular, p. 110; W. Z. Chien, Quart. appl. Math. 1, 297–327 (1944), in particular, p. 299.] The deformation due to a small displacement of the membrane is discussed, and stress-strain relations are given.

J. L. Synge, Ireland

**1463. E. F. Burmistrov, Symmetrical deformation of a nearly cylindrical shell** (in Russian), Prikl. Mat. Mekh. 13, 401–412 (1949).

The method of small parameters and the approximate Kirchhoff-Love theory of shells are used to study symmetric deformations of nearly cylindrical orthotropic shells of revolution with the radius of cross section of the form $r = a + \lambda f(x)$, where $\lambda$ is a small parameter. The main part of the paper is concerned with the deformation of shells with parabolic meridional section, given by $r = a[1 + \lambda(x^2/l^2 - 1)]$, $2l$ being the length of the shell. It is assumed that the shell is deformed by (a) uniform axial forces applied along the edge of the shell, (b) normal forces uniformly distributed over its lateral surface. In the application of the method of perturbations it is assumed that the displacements are of the form $u = u_0 + \lambda u_1 + \lambda^2 u_2$, and the stresses and moments are likewise given with sufficient accuracy, by similar expressions.

I. S. Sokolnikoff, USA

**1464. M. T. Huber, Bending of a curved tube of elliptic section,** Proc. seventh int. congr. appl. Mech. 1, 322–328 (1948).

This is a summary of the paper of Rev. 2, **1116.**

**1465. Giuseppe Grioli, On the deformation of a cylindrical shell with holes stressed uniformly** (in Italian), Pubbl. Ist. appl. Calc. no. 246, 5 pp. (1949) = L'Ingegnere 5 (May 1949).

The cylindrical components of the deformation of a cylinder of finite length, with evenly spaced circular holes, under uniform inside pressure are determined by the Rayleigh-Ritz method. The first two terms of the geometric series for the deformation components are taken into account in a numerical example, while the coefficients of the series are shown to be computable in terms of the Bessel function $J_1$.

Mario G. Salvadori, USA

**1466. G. Wästlund and S. G. A. Bergmann, Buckling of webs in deep steel I girders** (summaries in German and French), Publ. int. Ass. Bridge Struct. Engng. 8, 291–310 (1947).

Results are given for tests on plate-girder webs which were conducted at the Royal Institute of Technology in Stockholm. All tests were made on steel rectangular web plates welded to the flanges of girders. The investigation included three groups of tests: test series A in which the webs were subjected to shearing forces along all four edges; test series B in which the webs were subjected to bending moments acting on two opposite edges; and test series C in which the webs were subjected to shearing forces combined with bending moments. For each test specimen, measurements were made of the lateral deflection at various points of the web as a function of load, and strain-gage measurements were taken at some points on the web.

In all tests it was found that the webs deflected laterally from the very beginning of loading. In most cases the rate of increase in deflection was almost constant from zero load up to the load that caused yielding. The shapes of the deflection surfaces obtained in test series A and C showed fairly close agreement with those obtained from theoretical calculations. In test series B the agreement was not so satisfactory. The strain-gage measurements showed that, even at small loads, the state of stress in the webs differed greatly from those calculated on the assumption of plane webs.

With the exception of yield phenomena in weld seams, the first yielding in slender webs occurred in the middle of the waves due to buckling. This yielding was local in character and had little effect on the load-carrying capacity of the webs. The tests confirm conclusions of other investigators in showing that the assumptions involved in the linear theory of plates are not adequate to predict behavior of plate-girder webs. In particular it is again demonstrated that the theoretical critical load of plane web plates, calculated by linear theory, bears no direct relation to the ultimate load-carrying capacity.

A discussion is given of factors of safety against buckling of webs. A general design procedure for plate-girder webs is proposed which is based on the test results. Application of this proposed procedure would lead to savings of 10 to 15% in the weight of a web as compared with current design specifications.

This paper is a condensation of a 200-page report with the same title and by the same authors which was written in English and published in Stockholm.

Dana Young, USA

## Buckling Problems

(See also Revs. 1458, 1466, 1470)

**1467. Elbridge Z. Stowell, Plastic buckling of a long flat plate under combined shear and longitudinal compression,** Nat. adv. Comm. Aero. tech. Note no. 1990, 17 pp. (1949).

The author has studied previously the plastic buckling of a long flat plate elastically restrained along the edges, under longitudinal compression alone [same source, Rep. 898 (1948)] and under shear alone [same source, tech. Note 1681 (1948); Rev. 1, **1331**]. Using a theorem of Ilyushin, he shows that the solution may be obtained by the same method for any combination of these loadings, provided the ratio of shear to longitudinal compression is assumed constant during the loading process. The critical

load is evaluated approximately by means of the energy method described by the author in the first paper cited.

Combinations of shear and compression are computed for three simply supported plates of 24S-T4 aluminum alloy, reduced to stress-ratio form, and compared with the known interaction curve in the elastic region. Departures from this curve are found to be slight, provided the coordinates of the interaction curve are modified to allow for variation in moduli. Ch. Massonnet, Belgium

1468. **T. E. Schunk, The cylindrical shell strip above the buckling limit** (in German), Ing.-Arch. 16, 403–421 (1948).

The author investigates the state of stress and deformation in slightly curved cylindrical panels of infinite length, acted upon by normal and shear loads. The basic equations are taken in the form given by K. Marguerre [Proc. 5th int. Cong. appl. Math. 1938, pp. 93–101 (1939)] and approximate solutions are obtained by the direct methods of the calculus of variations (energy methods). Extensive numerical results are obtained for buckling loads, and for the stress distributions and deflection patterns due to loads beyond the buckling load. It seems to the reviewer that the work of this paper should be extended by a consideration of the possibility of the shell's buckling before the classical buckling load has been reached, in accordance with the work by Donnell, Cox, von Kármán and Tsien [J. Aeronaut. Sci. 8, 303–312 (1941)], and others. E. Reissner, USA

## Joints and Joining Methods

**(See Rev. 1466)**

## Structures

(See **also Revs. 1441, 1445, 1455, 1459, 1502, 1559**)

1469. **Miguel García Ortega, Framed structures** (in Spanish), Rev. Cienc. apli. 1, no. 1, 3–16 (Oct.-Dec. 1947).

This presentation consists in an explanation and expansion of an article, "Original procedure for the calculation of framed structures," by Francisco Franco in 1945, and it is assumed that the reader is familiar with that work. The author develops the equivalents of stiffness and carry-over factors for flexural members of nonconstant cross section. These, together with equations of the slope-deflection type, provide the starting point for the solution of a specific problem. Several examples are carried through the semitabular form of calculations employed.

Glenn Murphy, USA

1470. **W. H. Wittrick, A theoretical analysis of the efficiency of sandwich construction under compressive end load,** Rep. Memo. aero. Res. Counc. Lond. no. 2016, 20 pp. (Apr. 1945, issued in 1948).

This paper gives formulas for the optimum design of sandwich struts with free unloaded edges, and of circular cylinders under end load. Only three design parameters are considered, namely the core stiffness, the core thickness, and the skin thickness. All other quantities are assumed known.

The possible modes of failure for the sandwich strut are Euler buckling, skin wrinkling, and skin yielding. The three parameters are obtained by enforcing the conditions that the two instability failures should coincide, and then maximizing the strength to weight ratio. This leads to a skin stress which is less than the allowable within certain limits for the values of a parameter defined in the paper. Outside these limits the optimum design is obtained by making all three possible types of failure coincide.

Only two types of failure are considered for the sandwich cylinder, and the design condition of maximum strength to weight ratio is again employed. Curves are given for a given foam-core material and for duraluminum, steel or magnesium-alloy skins. The conclusion is drawn that the duraluminum skins are the most efficient with the present core materials, and that to use skin materials to their greatest advantage further development of very light foams is necessary. Bruno A. Boley, USA

1471. **W. Muckle, Experiments on a light alloy model superstructure,** N. E. Coast Instn. Engrs. Shipb. 65, 413–450 (May-June 1949).

An experiment is described in which a composite model structure consisting of two steel channels braced by welded vertical cross plates, and strengthened over the central part of the span by a superstructure of magnesium alloy consisting of zed beams and stiffened sheets, is subject to bending moments produced by vertical forces; the strains and deflections are measured. The model is designed to represent a ship structure in which the bridge is constructed of a lightweight alloy; the purpose of the experiment is the investigation of the longitudinal strength of such a ship as affected by the joint action of ship structure and bridge, produced by the riveting between these two parts.

The strain measurements indicate that the bending resistance of the composite model structure is about one third lower than that computed under the assumption of a fully effective composite moment of inertia. The reduction is probably due to a combination of the shear lag in the riveted joint of attachment of the bridge, the discontinuity at the ends of the superstructure, and local, mostly elastic, buckling of the light alloy sheets.

A. M. Freudenthal, USA

1472. **Paul Andersen, Substructure analysis and design,** New York, The Ronald Press Company, 1948, x + 305 pp., 216 figs. Cloth, 6.3 × 9.3 in., $4.50.

The author gives the conventional methods for the analysis and the design of substructures. The problems of the design engineer are emphasized, and practical examples are given. The book is intended for use in the field of structural engineering.

F. C. de Nie, Holland

## Rheology (Plastic, Viscoplastic Flow)

**(See also Revs. 1485, 1489)**

1473. **H. Brandenberger, New foundations of materials testing and of strength of materials** (in German), Schweiz. Bauztg. 65: 667–670 (Dec. 6, 1947); 681–685 (Dec. 13, 1947).

1474. **J. H. Palm, Stress-strain relations and necking criteria for triaxial loading, two principal stresses being equal,** Appl. sci. Res. Sec. A A1, no. 5–6, 353–377 (1949).

The author makes use of a previously derived expression for the uniaxial stress-strain properties of metals,

$$\sigma = \sigma_t - (\sigma_t - \sigma_c) \exp(-\epsilon/\epsilon_c),$$

where $\sigma$ is true stress, $\sigma_c$ is yield stress, $\epsilon$ is true strain and $\sigma_t$ and $\epsilon_c$ are constants, depending upon the stress-strain properties of the material. On the basis of the above formula and the assumption that the resistance to plastic flow depends only upon the shear stresses, he predicts the necking characteristics and the uniform strain at the time of necking for various combinations of uniaxial stress superposed upon a system of balanced transverse biaxial stresses. Both the conditions of constant transverse stresses and

variable axial stresses, and constant axial stresses and variable transverse stresses are treated. The condition for positive necking (local constriction), and negative necking (local bulge) are considered. It is concluded that negative necking is precluded by the fact that the condition for positive necking in one of the transverse directions is always reached first.

Other subjects discussed briefly are fracture, the pressure distribution in wire drawing, and the conditions associated with necking under combined hydraulic and axial loading.

William Schroeder, USA

**1475. H. Neuber, General solution of the problem of plane plasticity for a general isotropic or anisotropic law of plastic flow** (in German), Z. angew. Math. Mech. 28, no. 9, 253–257 (Sept. 1948).

A simple geometrical representation is given of the problem of plane plastic flow for an arbitrary plasticity condition. The plasticity condition, $f(\sigma_x, \sigma_y, \tau_{xy}) = 0$, is represented by a surface in a space in which $p = \sigma_x + \sigma_y$, $q = \sigma_x - \sigma_y$, and $t = 2\tau_{xy}$ are taken as rectangular cartesian coordinates. It is proved that the characteristics of the equations of equilibrium, $\partial\sigma_x/\partial x + \partial\tau_{xy}/\partial y = 0$ and $\partial\tau_{xy}/\partial x + \partial\sigma_y/\partial y = 0$, are represented by the two families of curves that lie on the surface $f(\sigma_x, \sigma_y, \tau_{xy}) = 0$, and which at any point $(p', q', t')$ are inclined at $\pm 45°$ to the plane $p = p'$.

H. G. Hopkins, England

**1476. N. F. Astbury, Some theoretical considerations on the dynamic properties of plastics,** Proc. roy. Soc. London 196, Ser. A, 92–105 (1949).

The dynamic behavior of plastic materials is considered, using for the stress-strain relationship a linear relationship between stress, rate of change of stress, strain, and rate of change of strain. This corresponds to the response to a stress increment consisting of an instantaneous elastic strain followed by the development of additional plastic strain based on a constant relaxation time. The configuration of standing plane waves in such a medium is investigated, by expressing the displacement in complex form involving a wave velocity and an attenuation constant, and choosing these to satisfy the equation of motion and the stress-strain relationship. The reflection and transmission of waves from a steady source with normal incidence on a plane slab of such material is investigated in the same way. The attenuation is shown to have a marked influence on the magnitude of the reflected wave.

E. H. Lee, USA

**1477. Albert Kochendörfer, Computation of deformation work of metals at ordinary and impact tensile strain** (in German), Z. Metallk. 39, no. 12, 376–384 (Dec. 1948).

This paper is the second one of a series in which an attempt is made to relate various mechanical properties of polycrystalline metals to the mechanical properties of the single crystals. In the first paper, published in Metallforschung [June 1947; Rev. 2, **1391**), formulas were derived for the tensile strength of face-centered cubic metals as a function of strain rate and temperature. In the present paper formulas for the work of deformation $A$ are derived for a uniform tensile specimen of cubic metal, and the results are found to be in approximate agreement with measured values for mild steel.

The derivation of $A$ proceeds by assuming a parabolic shape for the "true" stress-strain curve $\sigma_w(\delta) = \sigma_0 + (p\delta)^{1/2}$ where the yield strength $\sigma_0$ and the work-hardening parameter $p$ vary with temperature $T$ and strain rate $\omega$ as given in Rev. 2, **1391**. This leads to $A = \sigma_B - \sigma_0$ for ductile fracture, and

$$A = (\sigma_{wr} + \sigma_0)(\sigma_{wr}/\sigma_0 - 1)^2 \sigma_0^2/2p$$

for mixed ductile and brittle fracture, where $\sigma_B$, $\sigma_{wr}$ are the true stresses for maximum load and for fracture, respectively. The author gives a simple formula for computing $\sigma_B$ as a function of $\sigma_0$, $p$, and he assumes that $\sigma_{wr}$ has a constant value independent of temperature and rate of straining. Substituting these values he obtains theoretical transition curves $A(T)$ for copper and mild steel at rates of straining of $\omega = 1$ (7.5%/min) and $\omega = 10^4$ ($10^4 \times 7.5$%/min). The curves show the expected shift toward higher transition temperatures with increased speed of loading, and they show a much steeper transition curve for the mild steel than for the copper. The formulas also provide a simple means for computing the shift in transition curve as a function of rate of straining.

Walter Ramberg, USA

**1478. J. F. Nye, Plastic deformation of silver chloride. II. Photoelastic study of the internal stresses in glide packets,** Proc. roy. Soc. Lond. Ser. A 200, no. 1060, 47–66 (1949).

Rolled sheets of silver chloride were recrystallized and then plastically deformed. The main object was an investigation of the distribution of the stresses in the glide packets of favorably oriented crystal grains under polarized light after the sheets were plastically deformed. Nine birefringence patterns in glide packets of individual silver chloride grains are photographically reproduced for various orientations of the vibration directions in the Nicols relatively to the slip bands, and the patterns of dark lines explained.

A theory is advanced for the distribution of the stresses in the interior of a glide packet. Taking the $z$-axis normal to the plane of gliding, the stresses are resolved into their normal components $\sigma_x$, $\sigma_y$, $\sigma_z$ and shearing-stress components $\tau_{xy}$, $\tau_{yz}$, $\tau_{zx}$, consisting of a system of uniform stress such as $\sigma_x = \zeta_1$, $\sigma_y = \zeta_2$, $\sigma_z = \zeta_4$ and $\tau_{xy} = \zeta_3$, $\tau_{yz} = \zeta_5$, $\tau_{zx} = \zeta_6$ on which is superposed a system with linearly varying (bending) stresses: $\sigma_x = a_1 z$, $\sigma_y = a_2 z$, $\tau_{xy} = a_3 z$. Due to the slip along the boundary planes between adjacent packets, certain components of strain, and consequently of stress, must show discontinuities in these planes. Glide packets were furthermore found to have a curvature. Many useful observations concerning the mechanism of gliding and the residual stresses in the glide packets are described in detail.

A. Nádái, USA

**1479. H. S. Green, The kinetic theory of elasticity and viscosity in liquids,** Proc. int. rheolog. Congr. Holland, I: 12–28; III: 3–6 (1948).

This is a short expository article on the kinetic theory of liquids based upon the concept of the author and Born [Proc. roy. Soc. (A) 188, p. 10 (1946); 189, p. 103 (1947); 190, p. 455 (1947)], Kirkwood [J. chem. Phys. 14, p. 180 (1946)], and others, that the molecules are regarded as freely mobile. The basic quantities which enter the theory are the potential $\phi(r)$ between two molecules a distance $r$ apart, and the various frequency functions specifying the probability of various molecular distributions. The author first shows that the pressure in a liquid at rest is composed of two terms, the usual kinetic term due to molecular motion, and the potential term due to the potential energy of the molecules. This second term is important in liquids. Its computation involves the integration of the product $\phi'(r) r n_2(r)$ over all volume elements of $r$, and $n_2(r)$ is the probability of finding two molecules at a distance $r$. $n_2(r)$ is roughly

$$n_1^2 \exp(-\phi(r)/kT), \qquad (*)$$

$n_1$ being the number density and $T$ the temperature, but $n_2(r)$ is exactly the Fourier analysis of X-ray scattering picture. Computed isotherms verify the idea of van der Waals. The elastic

properties of the fluid are taken up next, and a formula is given for the shear modulus of the fluid. The difficult subject of viscosity is then discussed, showing that the important contribution comes not from the momentum transfer due to thermal motion of the molecules, but from the potential of molecules, and involves again $n_2(r)$. Because of (*), this leads qualitatively to the Andrade law [Phil. Mag. 17, pp. 497 and 698 (1934)] of decreasing viscosity with temperature. The quantitative theoretical computation is very difficult and is briefly discussed. The author then makes the interesting suggestion of directly measuring the pertinent frequency functions by X-ray scattering of a liquid under shearing motion. The paper concludes with a suggestion of how to treat the relaxation phenomena. H. S. Tsien, USA

1480. **Charles Sadron, Viscosity of macromolecular dilute solutions** (in French), Proc. int. rheolog. Congr. Holland, I: 62–94; III: 12–14 (1948).

The paper presents a review of the theories concerning the viscosity number $\eta_i = (\eta - \eta_0)/\eta_0 c$ ($\eta$ is viscosity of solution; $\eta_0$ is viscosity of solvent; $c$ is concentration) for dilute solutions of macromolecular substances (molecules well separated, containing between 100 and, say, 400,000 atoms); some comparisons with experimental results are discussed. In part II the formulas for rigid particles (spheres, ellipsoids of revolution, rods), without and with influence of Brownian movement, are given; attention is drawn to Simha's theory, leading to a higher influence of Brownian movement than was assumed by former investigators. Experimental results with tobacco-mosaic virus by Oster, Doty and Zimm are in favor of Simha's theory, but a satisfactory interpretation of all experimental data involved is difficult and the author does not consider the evidence to be conclusive. In part II the theories on flexible molecules (consisting of $N$ constituent parts) are considered. Attention is given to free and hindered rotation; the "rosary" type of molecule; the possibility for the molecule to coil up into a sphere with radius $\sim N^{1/2}$; possible porosity of the sphere. The author then gives an exposition of his own work on the theory of the "equivalent particle," which behaves as a rigid rod for $N$ small, $w_0/kT$ large ($w_0$ is a certain energy to be overcome in bending the link between two adjacent parts), with $\eta_i \sim N^2$; or as a sphere (radius $\sim N^{1/2}$; sphere assumed to be impenetrable) for $N$ large, $w_0/kT$ small, with $\eta_i \sim N^{1/2}$; in between there is a transitional domain. A comparison of calculated curves is made with a series of experimental results. A list of references is given, and also a summary of discussional remarks made after the paper was read. J. M. Burgers, Holland

## Failure, Mechanics of Solid State

**(See also Revs. 1461, 1474, 1477)**

1481. **Bekir Postacioğlu, Remarks on the existence-conditions of intrinsic curves** (in French, with Turkish summary), Bull. Tech. Univ. Istanbul 1, 36–47 (1948).

According to Mohr's theory the normal stress $\tau$ and the shearing stress $\sigma$ which will cause failure in an elastic material are connected by a continuous set of definite relations given by an intrinsic curve of the form $\tau = f(\sigma)$. Failure occurs when this curve touches the outermost of the Mohr circles. This curve can be obtained as an envelope of the family of circles $(x - p)^2 + y^2 = r^2$, where $p$ and $r$ are half the sum and difference of the principal stresses. Assuming that $r$ is a function of $p$ the author uses elementary methods to obtain the intrinsic curve. A linear relation between $r$ and $p$ gives good agreement with an empirical curve obtained in the case of concrete. This linear relation implies that, in the criterion of failure, besides the shearing stress, which is a primary factor in yield, volumetric stress of a particular type should also be taken into account. Such a law was formulated by J. J. Guest using various experimental results [Philos. Mag. 29, 289–297 (1940)]. B. R. Seth [Proc. Indian Acad. Sci. Sect. A. 14, 37–40 (1941)] deduced it as a direct consequence of the theory of finite deformation. B. R. Seth, USA

## Material Test Techniques

**(See also Revs. 1426, 1444, 1445, 1473, 1488, 1596, 1597, 1604, 1605)**

1482. **Kurt Matthaes, Semiautomatic strength testing of light metal sheets** (in German), Z. Metallk. 40, no. 5, 198–200 (May 1949).

A procedure is described for speeding up routine tensile tests of sheet metal in plant laboratories. The preparation of the specimens is speeded up by punching them out of sheet. The specimen is inserted between the grips of a tensile testing machine of the pendulum type. A lever-type strain gage of special design is attached to one side of the specimen (in disregard of good laboratory practice which dictates the use of gages on both sides of the sheet). The strain gage is provided with a contact point which opens up when the strain reaches a preset value. The test results for as many as ten specimens are written automatically on a chart in the form of pairs of colored lines the length of which is calibrated in terms of stress and of elongation, respectively. The writing of the "stress-line" is coupled to the deflection of the pendulum of the testing machine, and that of the "elongation line" is coupled to the displacement between grips. A blip is produced on the stress line by the opening of the contact on the strain gage when the offset strain is 0.2%. The procedure was used for testing 64,000 specimens at the Heinkel plant in 1943. Walter Ramberg, USA

1483. **Kurt Matthaes, Reception tests of light metal sheets** (in German), Z. Metallk. 40, no. 5, 196–197 (May 1949).

1484. **J. K. L. Andersen, Determination of an isothermal transformation diagram with an optical dilatometer,** J. Iron Steel Inst. 162, 29–32 (May 1949).

When most steels are heated to high temperatures there is a phase change or transformation that takes place at about 700 C. In certain alloy steels the reverse of this transformation takes place on slow continuous cooling only after the temperature has reached about 200 C. However if the cooling is interrupted and the specimen is held at some temperature above the lower transformation temperature, the transformation will gradually take place.

This paper contains the description of an optical dilatometer which has been made so that a time record of the elongation of the specimen during the transformation which is carried out at a constant temperature can be obtained. The time record is obtained by rotating the dilatometer at a constant speed in front of a sheet of photographic paper, thus obtaining a trace of the recording light beam. From the data obtained a plot of temperature vs. time for various percentages of total transformation can be produced. A study of the possible errors and their magnitudes is included in the paper. Evan A. Davis, USA

## Mechanical Properties of Specific Materials

**(See also Revs. 1453, 1496, 1497)**

1485. **T'ing-Sui Kê, Anelastic properties of iron,** Trans. Amer. Inst. min. metall. Engrs. 176, 448–478 (1948).

This paper, in preprint form, was covered in Rev. 2, **749**, the only new addition being discussions. J. L. Snoek notes the indication that carbon and nitrogen in cold-worked samples of iron are found either in the normal interstices or in a new preferential position at the edges of the dislocation. The author in reply mentions additional experimental work on stress-induced preferential diffusion in tantalum. J. M. Robertson, USA

**1486. F. Rapatz, H. Krainer, and K. Swoboda, Hardness and cutting properties of steels containing 9% to 13% Cr** (in German), Archiv. Eisenhüttenw. 20, no. 3-4, 115-123 (Mar.-Apr. 1949).

The authors present the results of extensive investigations in regard to the hardening techniques to be used with a number of steels with 9 to 13% chromium. The properties investigated were hardness and cutting performance. While special heat-treating procedure improved the cutting performance of the basic steel (1.75% C, 0.49 Si, 0.22% Mn, 13.04% Cr), only the addition of molybdenum, vanadium, and tungsten in percentages approaching those of high-speed tool steel gave cutting performances approaching that of high-speed tool steel.

Frank J. Mehringer, USA

**1487. W. Connert, E. Maurer, and R. Scherer, Tests on the suitability of chromium-vanadium steels for high-temperature, high-speed tools** (in German), Arch. Eisenhüttenw. 20, no. 5/6, 179-188 (May/June 1949).

This report shows many results from tests with different tool steels, primarily high-speed and high-temperature tools. The effects of different alloy members and different heat treatments upon brittleness and wear resistance are studied. Some recommendations for the choice of high-temperature tools are given.

Ragnar Nilson, Sweden

**1488. W. I. Pumphrey, Behavior of a chromium steel in the Jominy hardenability test,** J. Iron Steel Inst. 157, 27-30 (1947).

The increase in hardness at distances of 1.2 to 1.6 inches from the end of the Jominy-test bar, that had been observed with the title material, was investigated by an interrupted end-quench technique. This consisted in brine quenching after varying periods of time in the standard Jominy quenching fixture. Such treatment accentuated the humps in the hardness curve. Metallographic and electron microscope examination of structures permit determination of the S-curve for the material, and correlation with cooling rates at different positions along the bar.

Henry A. Lepper, Jr., USA

**1489. Joseph Marin, V. L. Dutton, and J. H. Faupel, Tests of spherical shells in the plastic range,** Weld. J. Res. Suppl. 13, no. 12, 15 pp. (Dec. 1948).

Spherical shells of semikilled steel conforming to ASTM specification A 70-40, welded together of two machined hemispherical shells of an inside diameter of $18^1/_4$ in. were subjected to internal oil pressure to rupture at temperatures from about +80 to −25 F. The strains were measured by SR-4 electrical gages. The same material was subjected to three types of control tests: static-tension, impact, and bend tests. A table gives a comparison of the actual strength and ductility values as obtained by internal-pressure tests at room temperature, and those theoretical values as calculated from the principal stress and from the shear condition using the values of the static-tension tests. It is significant that the true-ductility values of 73 to 89% obtained by the tension tests decrease to 27% at the internal-pressure tests with biaxial strain. Furthermore the tests show that, for the steel concerned, a decrease in temperature produces an increase in the combined-stress yield and ultimate strength, and a decrease in the combined-stress ductility. Although the change in ductility was small, the shells tested at −8 to 30 F fractured into a number of pieces. E. Siebel, Germany

**1490. J. M. Stevels, Progress in the theory of the physical properties of glass,** New York, Elsevier Publishing Co., Inc., 1948, xi + 104 pp. + 26 figs. + 22 tables. Paper, 5.5 × 7.7 in., $1.50.

This monograph is largely a survey of work which has been done in various parts of the world on some of the physical properties of glass. The period covered is mainly up to World War II. The work in Holland during the war is mentioned briefly, the author reporting that this phase of the research will be covered in the form of individual papers. The first chapter is a general introduction in which the nature of glass and its relation to solid and liquid structures are discussed. The ideas of Zachariasen are outlined and the structural investigations by means of X-rays of Warren are discussed. The second chapter is devoted to a review of the experimental and theoretical work on the density of glasses. Chapters 3 and 4 respectively deal with electrical conductivity and dielectric losses. The final chapter is on molecular refraction of glass. Regarding more recent work, an interesting account is given of the theory of Gevers and Du Pré (1947) on dielectric losses in which they assume a distribution of relaxation times corresponding to different excitation energies which are themselves assumed to be distributed in a Gaussian manner. This leads to a theoretical dependence of the power factor on temperature and frequency. Numerous references are given throughout the monograph. Edward Saibel, USA

**1491. Viktor Elsaesser, The mechanical phenomena of the spinning process of cuproammonium rayon A, B, C** (in German), Kolloid Z.: 111, 174-190 (Dec. 1948); 112, 120-141 (Feb.-Mar. 1949); 113, 37-53 (Apr. 1949).

The behavior of the viscosity of a plastic mass in a coagulating bath, and the behavior of the stress in this mass during its travel from the spinning nozzle to the funnel end, are studied in this three-part paper.

In the first part, after a brief description of the cuproammonium process, the author shows the possibility of determining the variation of the viscosity $\mu$ of the filament and the variation of its stress $\sigma$ in the funnel in terms of the distance $x$ to the spinneret, assuming eventually the laws of laminar motion for the relation between $\mu$ and $\sigma$, and for the friction between filament and water.

The methods of measuring the different quantities, with different spinning conditions, are then presented. The determination of the breaking strength could not be done with the accuracy desired by the author.

Results of a great number of experiments are given in the second part, including the influence of the spinning conditions on the viscosity curve $\log \mu = g(x)$, and the stress curve $\sigma = f(x)$.

The limitation of the spinning velocity caused by filament ruptures is explained. The decreasing relation between breaking strength and denier is given. From his experiments the author deduces that the micellar structure (irregular in section), on which the strength of the filament depends, is realized in an early coagulation state. The optimum coagulation state was investigated and found to have a mean viscosity of about 56,500 poise. Some properties of the filament for this optimum state are given.

In the third part it is shown that the irregular section structure of the optimum state becomes almost regular for the filament in the end section of the funnel. The strength limitation of the cuproammonium filament is finally shown to be a result of the friction between water and filament, so that, with some proposed means for eliminating this friction, strengths up to 5 gr/denier

could be obtained, and that with pure mechanical methods a well-oriented and dense structure of cuproammonium filament can be obtained. D. De Meulemeester, Belgium

1492. **L. S. Martz, Preliminary report of developments in interrupted surface finishes,** Proc. Instn. mech. Engrs. 161, W.E.P. no. 47, 1–9 (1949).

The article describes interrupted surface finishes in connection with sliding-friction performance of bearings under both boundary and hydrodynamic lubrication. Description is given of the three main methods for obtaining interrupted surfaces: mechanical, chemical, and electrical, which may be of pitted, channelled, and porous characteristics. The author indicates possible specifications of interrupted bearing surfaces for the above two types of lubrication. The article contains 29 very good illustrations of surface finish. L. M. Tichvinsky, USA

1493. **F. L. Fitzpatrick and W. Serkin, Effect of mixing sequence on the properties of concrete,** J. Amer. Concr. Inst. 21, no. 2, 137–140 (Oct. 1949).

Our work seems to have established that when concrete is made by first mixing the fines (sand) and cement, then adding water to make a mortar, and finally incorporating the coarse aggregate into that mortar, significant improvement in concrete quality is achieved. Compared with concrete made by the customary method of charging aggregates and cement, and then adding water, there are substantial favorable differences in density, workability, strength, absorption and surface finish (and probably in permeability and durability). Authors' summary

## Mechanics of Forming and Cutting

(See also Revs. 1474, 1486, 1487, 1491, 1492)

1494. **I. M. Pavlov, On the question of interaction of the tool and the plastically deformable body** (in Russian), Izv. Akad. Nauk SSSR. Ser. tekh. Nauk 1, 85–99 (Jan. 1949).

The paper examines the various modes of interaction which can occur between tool and material in the course of forming processes. The actions of the tool are subdivided into: (1) active and passive, according to whether the strain is of the same or opposite sign as the stress (thus, if $0 < \sigma_2 < {}^1/_2(\sigma_1 + \sigma_2)$, and $\epsilon < 0$, the action is passive); (2) reversible or irreversible (thus a wall can only produce compressive stress); (3) rigid or flexible, according to the resistance of the tool.

There is no indication of how these various modes of action influence the forming process itself. D. Rosenthal, USA

1495. **George S. Mican, Deformation in rolling,** Iron Steel Engr. 26, 53–67 (Feb. 1949).

The changes developed under rolling or forging are shown to be related to the ratio of the height of stock worked to the horizontal component of the chord of the roll contact arc, or the die width, respectively. When subjected to the same reduction in both cases, cross-sectional contour patterns, metal-flow patterns, dimensional extensions, induced stresses, and quality effects are substantially equal. The cross-section contour changes from one with convex side walls to one with concave side walls when stock height to the ratio is increased, the critical ratio lying at about 2 for all grades of steel. Rolling stresses in the middle plane will therefore be extensional for all ratios greater than 2.2, compressive for all ratios below 1.8. Specimens built up of thin disks revealed the same deformations and allowed a study of metal flow. The disks in contact with the die surfaces are not deformed to the same extent as the disks farther removed from the surfaces. Several of the disks' peripheries move completely into the die contact surfaces. The occurrence of edge seams and cracks is explained. At sufficiently high rolling speed, differences between rolling and step forging are produced by reason of friction between the roll surfaces and the stock, resulting in slightly greater elongations and lesser spread attained in rolling as compared with step forging. A number of cases of special profiles are discussed. C. Zwikker, Holland

1496. **J. P. Doan and G. Ansel, Some effects of zirconium on extrusion properties of magnesium-base alloys containing zinc,** Trans. Amer. Inst. min. metall. Engrs. 171, 286–305 (1947).

The magnesium-rich corner of the Mg-Zn-Zr system has been studied in the composition range 0 to 7% Zn and 0 to 1% Zr with regard to extrudability, tension and compression, metallography and toughness. The alloys investigated appear destined for commercial use as extrusion alloys. From authors' summary

1497. **D. R. Cox, Theory of drafting of wood slivers,** Proc. roy. Soc. Lond. Ser. A, 197, 28–51 (May 1949).

The author's purpose is to make a mathematical study of the dynamical mechanism of sliver drafting, with the object of relating the changes of sliver thickness (being expressed by the number of fibers in cross section) to the physical properties of the sliver and of the drafting mechanism by relating them both to the statistics of fiber motion.

He has given drafting equations for a very idealized model of a sliver, with a detailed solution only for the special case when the fiber length is small compared with the length drafted(mule drafting) or with the distance between rollers (roller drafting).

Under this hypothesis it is shown that in the mule drafting, in the general case, no drafting wave is obtained while in roller drafting. From some additional assumptions, it appears that a drafting wave is generally formed when the sliver is drafted at constant roller velocity, and this tendency increases by increasing draft.

The author is first to acknowledge that the assumptions on which the present paper are based are very simplified and require both extension and deeper analysis. Further work, in which he hopes to deal in detail with drafting when the fiber length is not small, and also to give a deeper analysis of the principles underlying the motion of the fibers, shall be expected with very great interest. D. De Meulemeester, Belgium

## Hydraulics; Cavitation; Transport

(See also Revs. 1506, 1532, 1556, 1570, 1581, 1583)

1498. **Olaf P. Bergelin and Carl Gazley, Jr., Co-current gas-liquid flow I: Flow in horizontal tubes,** Heat Transf. fluid Mech. Inst., 5–18 (May 1949).

This paper describes the characteristics of gas-liquid flow in horizontal tubes. The major portion of the paper is devoted to the characteristics of stratified flow. Laminar and turbulent flow in each phase are discussed and the conditions of interfacial wave formation are given. The gas-phase pressure drop along a smooth interface is shown to be similar to the pressure drop during single-phase flow in a smooth tube, while the pressure drop along a wavy interface exhibits the characteristics of flow in a rough pipe. John A. Lewis, USA

1499. **O. P. Bergelin, P. K. Kegel, F. G. Carpenter, and Carl Gazley, Jr., Co-current gas-liquid flow II: Flow in vertical tubes,** Heat Transf. fluid Mech. Inst., 19–28 (May 1949).

An experimental study is reported of the pressure drop for the

downward turbulent flow of air and water through a vertical 1-in. tube. The results are compared, on a friction-factor-Reynolds-number basis, with data for annular flow in a horizontal tube, and the two cases are shown to approach each other at high gas velocities. In both cases the presence of the liquid phase causes "rough-pipe" pressure-drop characteristics for the gas phase. The experimental results are extended to cover the pressure drop during the condensation of water and several organic vapors in a vertical tube condenser. An empirical relation is proposed as a correlating factor for the physical properties of the materials, and a tentative correlation is presented. Measured and calculated pressure drops are compared for the condensation of five materials. John A. Lewis, USA

**1500. Carl Gazley, Jr., Co-current gas-liquid flow III: Interfacial shear and stability,** Heat Transf. fluid Mech. Inst., 29–40 (May 1949).

In order to evaluate the energy losses, energy transfers, and exchange phenomena at a fluid-fluid interface, the data previously reported for horizontal stratified flow are used to evaluate interfacial shear and stability. Comparison of the data with those obtained in absorption columns and at air-sea interfaces indicates that at the point of interfacial instability the energy losses and transfer rates at the interface increase sharply. It is also indicated that the formation of interfacial waves is essentially dependent only upon the liquid depth and the relative velocity of the two fluid streams. John A. Lewis, USA

**1501. R. Bolant, Probabilistic determination of permissible discharges in hydraulic networks** (in French with English summary), Houille blanche 4, 315–330 (May-June 1949).

The author endeavors to place the design of water supply systems upon a rational basis. To accomplish this it is assumed that demands are intermittent and random, and that the system is satisfactory if it supplies the demands $100m\%$ of the time, $m$ being called the "figure of merit." Methods of determining $m$ for given systems are based upon standard probability theory. Two examples, one for drainage from a battery of lavatories, and one for supply to a series of machines, are solved. Two additional applications of the method to more complex systems are given. The methods suggested are applicable both to industrial and municipal problems of water supply and drainage. Their success is dependent upon the reliability of the initial data upon which they are based. In addition to the very readable French text, there is a good English synopsis which is sufficiently extensive to permit combination with the equations of the text by one unable to read the original text. Dwight F. Gunder, USA

**1502. R. R. Minikin, Floating and foundationless breakwaters,** Engineering 166: no. 4325, 577–579; no. 4326, 604–605 (Dec. 1948).

Prefabricated fixed and floating breakwaters are discussed from a practical point of view. The fixed breakwaters discussed are of the type that is fabricated principally of reinforced concrete, floated into place and sunk without any elaborate preparation for foundations. Examples are given of the stability analysis of a breakwater with a vertical face using formulas of Sanflou and of the author for the wave forces.

An analysis is made of the mechanism by which a floating breakwater protects against wave action, and the experience in the Normandy landings of World War II with the Bombardon floating breakwater which had a cruciform-shaped cross section is described. Vito A. Vanoni, USA

**1503. Waterways Experiment Station, Conduits and Howell-Bunger valves, Narrows Dam, Little Missouri River, Arkansas; model investigation,** Wways. exp. sta. tech. memo. no. 2-294, 41 pp. (Aug. 1949).

Tests conducted on a 1:16-scale model revealed the necessity of (1) a hood for the Howell-Bunger valves to prevent flow from being projected onto the access road along the right side of the stilling basin; and (2) replacement of a sloping end sill by a vertical-faced sill. Eben Vey, USA

## Incompressible Flow: Laminar; Viscous

(See also Revs. 1417, 1418, 1479, 1480, 1567, 1606, 1607, 1609, 1611, 1614)

**1504. Antonio Pignedoli, On cylindrical vortices** (in Italian), Atti Semin. mat. fis. Univ. Modena 3, 102–124 (1948–1949).

This paper concerns the two-dimensional motion due to a single cylindrical vortex rotating with constant angular velocity $\omega$ in inviscid incompressible fluid at rest at infinity. Let $C$ denote the cross section of the cylinder, $\psi$ the stream function, and $\zeta$ the vorticity at the point $(x, y)$ referred to axes rotating with $C$. If $\zeta$ is constant, we have a homogeneous vortex; if $\zeta$ is variable, a nonhomogeneous vortex. Assuming continuity of pressure and velocity at the boundary $C$, it is shown that the nonhomogeneous case has, in general, no solution when $\zeta$ is arbitrarily assigned. The author then proceeds to solve completely (i) the nonhomogeneous case in which $C$ is a circle and $\zeta$ is a linear function of $\psi + {}^1/_2\, \omega(x^2 + y^2)$, and (ii) the homogeneous case in which $C$ is a closed epicycloid. Both these solutions are new to the reviewer. L. M. Milne-Thomson, England

**1505. Luisa Fabbrichesi, Stability of a rigid configuration of four vortex threads** (in Italian), Ist. Veneto Sci. Lett. Arti. Parte II. Cl. Sci. Mat. Nat. 106, 67–74 (1948).

A steady configuration of vortexes at the vertexes of a rhombus, discovered by Laura [same source, 97, 535–540, 813–818 (1938)] is found to be stable if, and only if, the ratio of the lengths of the diagonals is less than $\sqrt{(2 + \sqrt{3})}$. M. J. Lighthill, England

**1506. L. Carlotti, Contribution to the study of swell in the vicinity of shores** (in French), Houille blanche: no. 6, 469–480 (Nov.-Dec. 1947); no. 2, 193–202 (Mar.-Apr. 1948).

The paper analyzes the present knowledge on the swell on seas with limited depth, and works out the fundamental laws for model tests on harbors and coastal work. The similarity of shape is realized when two photographs, of model and of prototype, taken under similar circumstances, can be superimposed by choosing convenient model scales. The approximate solution of the swell (Poisson's theory) leads to the formula:

$$T = \Lambda/[(g\ \Lambda/2\pi)\tanh(2\pi H/\Lambda)]^{1/2} \qquad (1)$$

where $T$ is the period of the swell, $\Lambda$ the wave length and $H$ the depth of the sea in nature. The same small letters apply to the model. The model scales to be introduced are the scale for the lengths $\alpha = \lambda/\Lambda = l/L$, the scale for the depth $\beta = h/H$ and the time scale $\theta = t/T$. The old model technique used to write approximately $T = \Lambda/\sqrt{gH}$ instead of (1) so that $\theta = \alpha\sqrt{1/\beta}$. According to the author, this is possible only when $(2\pi H/\lambda)$ is very small.

According to the new method proposed, a value $T_0$ of $T$ has to be chosen and similarity will be achieved for this sole value. The model scales $\alpha$ and $\theta$ will then be given by (1) together with

$$\tanh(2\pi h/\alpha\Lambda) = (\alpha/\theta^2)\tanh(2\pi H/\Lambda).$$

The second part of the paper deals with the damping of the oscillations in nature and on the model. The effect of the variations of width and of depth, of resonance effects and of the friction, are analyzed and the laws of similarity are established and compared with results from models. Charles Jaeger, England

1507. **F. Riegels, Incompressible potential flow past given obstacles, I** (in German), Ing.-Arch. 16, 373–376 (1948).

In the case of a symmetric profile at no incidence, the potential problem would be solved by mapping the profile in the $z$-plane on a slit in a $\zeta$-plane, and distributing sources along the slit to satisfy the boundary condition. The derivative of the transformation at points of the contour is needed to calculate the surface velocity; its value is $ds/d\xi$, where $ds$ denotes the element of arc length of the profile and the slit lies on the $\xi$-axis ($\zeta = \xi + i\eta$). The author proposes to use the approximate relation $ds/d\xi = cds/dx$, $c$ = constant, where $z = x + iy$. This leads to simple formulas for the surface velocity distribution and seems to be a good approximation for sufficiently thin profiles. Two examples are shown. W. R. Sears, USA

1508. **John S. McNown and En-Yun-Hsu, Pressure distributions from theoretical approximations of the flow pattern,** Heat Transf. fluid Mech. Inst. 65–76 (May 1949).

The authors review the fundamental principles of the representation of cylindrical and axisymmetric irrotational motions. They then show how approximate graphical methods indicated by several authors, and chiefly by von Kármán, yield, for a correct distribution of vortexes, sources, and doublets, the boundary profile of flow and also the distribution of velocities and pressures around various obstacles. Comparison with experience shows agreement sufficient for practical applications. The authors consider these methods preferable to experimental studies which they regard as more expensive. L. Escande, France

1509. **E. Souczek, Airfoils in nonhomogeneous flow** (in German), Öst. Ingen.-Arch. 3, no. 5, 396–404 (1949).

This paper gives an extension of the theory of thin airfoil profiles developed by Birnbaum and Glauert, assuming a certain vortex-distribution along the center line of the profile. A properly chosen distribution allows a good approximation of the boundary conditions for a thin, slightly curved profile in an arbitrarily inhomogeneous original flow. The lift distribution, circulation, lift, and the moment with respect to the leading edge are thereby computed.

In the general theory, the shape of the profile, the distribution of the angles of attack, and the distribution of velocity are represented by Fourier series. For practical purposes, an approximate method using trigonometric polynomials of three terms is indicated. W. C. Johnson, Jr., USA

1510. **K. Jaeckel, A simplified derivation of Weissinger's circulation equation for a yawed wing** (in German), Veröff. math. Inst. Tech. Hochschule Braunschweig 1947, no. 8, i + 5 pp. (1947).

Reconsidering the problem of a yawed lifting line, this author notes that the downwash induced just above the line can be written as the sum of five terms, of which one vanishes, and only one becomes infinite (logarithmically) at the line. He proposes to calculate this singular term approximately by distributing the corresponding increment of circulation over the chord in a certain fashion and using plane-flow formulas. This results, finally, in an integral equation for the total circulation distribution that is the same as Weissinger's [Math. Nachr. 2, 45–106 (1949); Rev. 3, **917**; also Nat. adv. Comm. Aero. tech. Memo. no. 1120 (1947); Rev. 1, **509**] although the derivation is different. [For another point of view see Reissner, Proc. nat. Acad. Sci. Wash. 35, 208–215 (1949).] W. R. Sears, USA

1511. **N. F. Sakharnîĭ, Flow around of a system of two profiles of given form** (in Russian), Prikl. Mat. Mekh. 13, no. 4, 445–448 (1949).

The incompressible flow around a biplane of infinite aspect ratio with parallel chords $b_1$ and $b_2$ is tackled by the conventional Glauert-type thin-airfoil method. The key simplification consists in approximating the distance effect in the cross-influence terms in the standard (coupled) integral equations by a best-fitting cubic. The leading Fourier coefficients in the expansions for the vorticity distributed along the chords are then determinable from four linear equations in four unknowns, rather than from an infinite system of equations. M. V. Morkovin, USA

1512. **Arthur W. Goldstein and Meyer Jerison, Isolated and cascade airfoils with prescribed velocity distribution,** Nat. adv. Comm. Aero. Rep. no. 869, 15 pp. (1949).

The problem of designing an airfoil developing a prescribed velocity distribution in a uniform stream is considered. An approximate shape is first assumed with a vortex distribution over its surface corresponding to the given velocity distribution. The values of the stream function at points on the surface due to the vortex distribution and the uniform stream are then calculated. The departure from constancy of these values is then a measure of the closeness of the assumed shape to the correct shape. Formulas are then developed, involving these initial variations of the stream function from constancy at various points of the airfoil, for an angular rotation of the airfoil and a modification of its shape which should tend to minimize the departure of the stream function from a constant value. The process can then be repeated with the new shape, and so on, until the stream function over the airfoil is satisfactorily constant and the variation of shape from one stage to the next is acceptably small. The paper does not indicate what conditions must be imposed on the prescribed velocity distribution to ensure a resulting real shape; these are presumably left to be dictated by experience. The method is readily extended to the design of airfoils in cascade with prescribed velocity distributions. Tables are given of the influence functions of a unit vortex and of an infinite row of unit vortices which are required in the application of the method to the case of an isolated airfoil and a cascade of airfoils, respectively.

A. D. Young, England

1513. **E. J. Richards and C. H. Burge, An aerofoil designed to give laminar flow over the whole surface with boundary-layer suction,** Rep. Memo. aero. Res. Counc. Lond. no. 2263, 12 pp. (1949).

A type of airfoil is described over which falling pressures are maintained over the whole surface except at a single point at which abrupt pressure recovery is made. With this airfoil employing a suction slot at the abrupt-pressure-rise position, it was anticipated that laminar flow could be maintained over the entire airfoil surface. Preliminary small-scale experiment at a Reynolds number of $0.37 \times 10^6$ showed that with a single suction slot at the abrupt-pressure-rise position, laminar flow could be maintained over the whole surface but an excessive amount of air had to be removed through the suction slot. Employing two suction slots, one at the abrupt-pressure-rise position and another 5% of the chord forward of this position, gave complete laminar flow requiring only 70% of the boundary-layer air to be removed by

suction. The effective drag coefficient of the test airfoil was not low, but it was anticipated that at flight Reynolds numbers a very low drag coefficient might be attainable.

H. Julian Allen, USA

**1514. Mahmoud Ali Hassan, Application of the electrolytic method to the Betz theory of clearance losses in blade cascades** (in German), Mitt. Inst. Aerodyn. Eidgenöss. tech. H. Zurich no. 15, 73 pp. (1949).

In various fluid machines there is a clearance between the edges of the rotating blades and a fixed wall. This paper presents a technique for determining the induced loss in this clearance space for two-dimensional frictionless incompressible flow. The technique uses the analogy between potential fluid flow and electrical flow in an electrolyte in a tank.

A comparison is given of the analytic solution for flow around a wall of rectangular contour in a parallel stream, and the electrolytic solution. A discussion is given of the theory of the induced loss in the clearance space. The analytic solution for flow around walls with induced cross flow compares well with the electrolytic solution. Numerical results are presented.

R. C. Binder, USA

**1515. P. Mandl, Two-dimensional hinge moment calculations,** Nat. Res. Counc. Canada aero. Rep. no. MA-214, 1–19, A-1 to A-6, 14 figs. (Oct. 1948).

An attempt is made to develop an analytic procedure for calculating two-dimensional hinge moments that takes into account the presence and size of the boundary layer. The technique applied was to use the boundary-layer thickness to locate a potential-flow streamline along which the pressure distribution assumed to be acting on the airfoil surface was calculated. The method involved a large number of assumptions and approximations, and the results were not altogether satisfactory. The author, consequently, suggests a number of ways for improving the method.

Arthur L. Jones, USA

**1516. H. C. Brinkman, Calculations on the flow of heterogeneous mixtures through porous media,** Appl. sci. Res. Sec. A, A1, no. 5–6, 333–346 (1949).

Muskat and Meres [Physics 7, p. 346 (1936)] have formulated basic differential equations governing the motion of heterogeneous fluid mixtures through porous media. They obtained a solution of these equations by numerical integration for the problem of a column initially filled with liquid saturated with gas, which is closed at one end and kept at a constant low pressure at the other.

Buckley and Leverett [Trans. Amer. Inst. min. metall. Engrs. 146, p. 107 (1942)] obtained an analytical solution for the problem of a column initially filled with liquid which is flooded with a second immiscible liquid. They found a solution in which the saturation is a three-valued function of the coordinate along the column.

In our paper in the first place a discussion is given of Buckley and Leverett's solution. It appeared that the true solution which contains a discontinuity may be derived from the three-valued solution by a discussion of the integral relation which represents the total liquid recovery from the column. This discussion bears a formal resemblance to that occurring in the theory of van der Waals' equation of state.

The second problem treated in our paper is that of a vertical column initially filled with liquid saturated with gas under a high pressure, which is opened at its lower end. For this problem we succeeded in finding an analytical solution for low values of the pressure gradient. Here again the paradox of a three-valued solution occurred and led to a discontinuity in the saturation.

A general argument is given to the effect that three-valued solutions are unavoidable for a theoretical treatment based on Muskat's equations of problems which lead to discontinuities in the saturation.

Author's summary

**1517. F. C. Gunther and Frank Kreith, Photographic study of bubble formation in heat transfer to subcooled water,** Heat Trans. fluid Mech. Inst., 113-126 (May 1949).

The authors have applied high-speed flash photography to the study of the formation and collapse of bubbles when a pool of subcooled water is subjected to a heat flux of up to 4 Btu per sq in per sec. A theory is put forward to account for the behavior observed. An error has crept into equation (2) in regard to the liquid density.

R. E. Button, Australia

**1518. A. Signorini, Theoretical determination of the dimensions of a special type of floating reservoirs** (in Italian), Pubbl. Ist. appl. Calc. no. 211, 91–112 (1948) = Rev. Tucuman (1948).

The paper deals with the dimensioning of gasoline storage tanks floating in the sea.

Ed.

## Compressible Flow, Gas Dynamics

(**See also Revs. 1543, 1548, 1550, 1564, 1565, 1568, 1570, 1572, 1574, 1583**)

**1519. Antonio Romero Juarez, Propagation of waves of finite amplitude** (in Spanish), Comisión Impulsora y Coordinadora de la Investigación Científica (Mexico), Anuario 1947, 97–103 (1949).

The author considers the one-dimensional equation of motion (Eulerian form) $y_{tt} = -\rho^{-1}p_y$ under the assumption that $p/\rho^\gamma = p_0/\rho_0^\gamma$. Employing the continuity equation, and using Lagrangian variables $x$ and $t$, the equation of motion becomes $\rho_0 p_0^{-1}\gamma^{-1}y_{tt} = y_x^{-\gamma-1}y_{xx}$. Two particular solutions (waves of finite amplitude) of this equation are discussed.

*Courtesy of Mathematical Reviews* J. B. Diaz, USA

**1520. Lipman Bers, An existence theorem in two-dimensional gas dynamics,** Proc. Symposia appl. Math. 1, 41–46. American Mathematical Society, New York (1949).

The following existence theorem is proved. Let there be given a profile $P$ and a point $z_T$ on $P$. Let $P$ be convex and continuously curved, except, perhaps, for a sharp trailing edge at $z_T$. For every angle of attack and every value of the maximum local speed there exists a potential gas flow around $P$ which is uniform at infinity, satisfies the Chaplygin density-speed relation, and possesses at $z_T$ a stagnation point. Here the Chaplygin relation referred to is that corresponding to $\gamma = -1$, where $\gamma$ is the ratio of specific heats. The proof is given for symmetric flows, but it is stated that the extension to asymmetric cases can be made. The method used here is to relate the problem to the solution of a certain integral equation and to apply a theorem of Leray and Schauder [Ann. Sci. École Norm. Sup. 51, 45–78 (1934)]. It is also shown that potential flow of a Chaplygin gas past a convex corner is impossible. In a note added in proof, the author states that he can replace the condition that $P$ is convex by a less restrictive condition.

W. R. Sears, USA

**1521. Stefan Bergman, Operator methods in the theory of compressible fluids,** Proc. Symp. appl. Math. 1, 19–40 (1949).

This is a further paper in a series concerning the solution of the hodograph equation of two-dimensional flow by the author's method of integral operators [Rev. 3, 751]. Methods of con-

structing solutions representing mixed flows are indicated; in particular, it is shown how solutions different from those of Chaplygin may be generated. The author also outlines a method, based on the application of integral operators to orthogonal functions, for solving nonlinear differential equations by a direct method of successive approximation.

Maurice Holt, England

1522. **Manfred Schäfer, On the calculation of the propagation of disturbances in a supersonic flow by direct hodograph method** (in German), Z. angew. Math. Mech. 29, 5–6 (Jan.-Feb. 1949).

The characteristics (Mach lines) are the "carriers" of discontinuities of velocity derivatives. If the effects of a boundary are treated as small perturbations of a known flow, one may approximate by neglecting their effect on the Mach waves themselves, i.e., use the characteristics of the unperturbed flow. This approximation has been made for the case of perturbed circulatory flow around a circular cylinder, whch was studied by G. I. Taylor [same source, 10, 334–345 (1930)]. The author states that his results are in qualitative agreement with Taylor's.

W. R. Sears, USA

1523. **N. Coburn and C. L. Dolph, The method of characteristics in the three-dimensional stationary supersonic flow of a compressible gas,** Proc. Symposia appl. Math. 1, 55–66. American Mathematical Society, New York (1949).

This paper is an exposition and development of the three-dimensional method of characteristics following an attack similar to that of E. Titt [Ann. Math. 40, 862–891 (1939)]. The point of view is generally that of differential geometry, and tensor notation is used consistently throughout. A curvilinear coordinate system is established using two families of characteristic surfaces plus a third family of surfaces whose members pass through the intersections of the characteristic surfaces. The equations of motion of a steady supersonic isentropic irrotational flow are expressed as 8 nonlinear first-order partial differential equations for the space variables and the velocity components. For the particular case where the intersections of the characteristic surfaces lie in the equipotential surfaces, the equations are simplified and conditions are found for solutions analogous to those for two-dimensional simple waves, two-dimensional and axially symmetric flows appearing as special cases. No practical examples are given.

W. D. Hayes, USA

1524. **E. Krahn, An approximate method for the calculation of compressible subsonic flow** (in German), Z. angew. Math. Mech. 29, 2–3 (Jan.-Feb. 1949).

As a first approximation, in plane flow, it is assumed that $\mathbf{v}/v_\infty = (\mathbf{V}/V_\infty)\,(\rho_\infty/\rho)^{1/2}$, where $\mathbf{v}$, $\mathbf{V}$ are the velocity vectors and $v_\infty$, $V_\infty$ the stream speeds in compressible and incompressible flow past the same profile, $\rho$ denotes the density, and $\rho_\infty$ its value in the undisturbed stream. For a better approximation the author writes

$$\frac{\mathbf{v}}{v_\infty} = \frac{\mathbf{V} + \mathbf{v}'}{V_\infty}\,\frac{\rho_\infty^{1/2}}{\rho^{1/2}},$$

and assumes that $\mathbf{v}'$ is a small correction vector. The equations of continuity and irrotationality lead to expressions for div $\mathbf{v}'$ and curl $\mathbf{v}'$, and finally to

$$2\partial v'/\partial z = m^2\lambda \bar{V}\, \partial|V|^2/\partial\bar{z} \qquad (1)$$

where $z$, $\bar{z}$ are the usual complex coordinates, $v'$, $V$ are the complex equivalents of $\mathbf{v}'/v_\infty$, $(\mathbf{V} + \mathbf{v}')/V_\infty$, $m$ is a constant determined by the stream Mach number, and $\lambda$ depends on $m|V|$. The author's method is to neglect $\mathbf{v}'$ in $V$, i.e., on the right side of (1). This permits calculation of $v'$ from the incompressible flow. He states that he has done this for a large number of profiles and for elliptic cylinders, for a range of Mach numbers. No results are presented here, nor is there any indication of the valid range of Mach numbers for this method.

W. R. Sears, USA

1525. **Klaus Oswatitsch, Laws in transonic flow** (in German), Z. angew. Math. Mech. 29, 4–5 (Jan.-Feb. 1949).

The transonic similitude law discovered independently by von Kármán and Guderley is reviewed briefly and is contrasted with other approximations. It is pointed out that the available pressure-correction formulas err in predicting the minimum-pressure point too far forward. A new formula (apparently purely empirical) is given to compute the downstream displacement of the pressure curve as function of pressure coefficient. It is stated that the corresponding drag increment is in good agreement with experiment.

W. R. Sears, USA

1526. **Y. H. Kuo, On the stability of transonic flows,** Proc. Symposia appl. Math. 1, 72–73. American Mathematical Society, New York (1949).

This is a preliminary report of an investigation of the stability of two-dimensional compressible potential flow involving an imbedded supersonic region. A flow past a thin body is assumed, for which the velocity potential is expanded in a power series in the thickness parameter. The time-dependent disturbance assumed is a triangular pulse, moving upstream, its head consisting of a weak shock wave. It is proposed to trace the history of this pulse, in terms of the velocity of its head wave and the slope of its tail.

W. R. Sears, USA

1527. **Fr. Vandrey, Reflection of weak disturbances at discontinuity surfaces of a plane subsonic flow** (in German), Z. angew. Math. Mech. 29, 1–2 (Jan.-Feb. 1949).

A plane subsonic stream is bounded by two walls, parallel except for small distortion of the lower one. The stream itself consists of two parts, having undisturbed speeds $U_0$ (lower) and $U_1$ (upper), separated by a discontinuity surface. The problem is to determine the effect of the small distortion of the bottom wall on the shape of the discontinuity-surface, and ultimately the effect of the upper boundary on the lower flow field. It is pointed out that this effect is opposite in sign for $U_1 = 0$ and $U_1 = U_0$, respectively, so that there is presumably a value of $U_1$ for which the influence of the upper wall is zero. The problem is treated by means of the linear-perturbation theory, the potential being expressed as a Fourier series or integral. The case of vanishing wall effect is found. [For certain related investigations see Howarth, Proc. Camb. phil. Soc. 44, 380–390 (1948), and Tsien and Finston, J. aero. Sci. 16, 515–528 (1949).]

W. R. Sears, USA

1528. **R. Sauer, Shock waves in one-dimensional unsteady gas flow** (in German), Helv. phys. Acta 22, 467–472 (1949).

A short introduction to the treatment of unsteady one-dimensional shocks by a graphical method.

Gottfried Guderley, USA

1529. **Stuart R. Brinkley, Jr. and John G. Kirkwood, Theory of the propagation of shock waves from cylindrical charges of explosive,** Proc. Symposia appl. Math. 1, 48–54. American Mathematical Society, New York (1949).

The authors consider explosion waves from cylindrical charges when (a) the explosion proceeds radially from the center line of

the charge, and (b) the explosion wave travels axially down the charge. Using the Euler equations of motion and continuity, the Rankine-Hugoniot shock equations, the energy equation (which allows for entropy change in the medium as the shock passes) and an arbitrary equation of state, the exact differential equations are formulated. The approximation used consists in replacing the energy-time relationship, which actually depends on integrals of the equations, by an explicit approximation which is adequate for most practical purposes. When this is done in case (a), there results a pair of ordinary differential equations for peak pressure and energy at the shock as a function of the radial coordinate $R$. The equations involve functions which can be tabulated from the exact thermodynamical properties of the medium. An essentially similar result is given for case (b). Here the solution is again a function of time and the radial coordinate only, since the radial and axial coordinates are connected by the equation of the surface of revolution formed by the shock wave in the exterior medium. D. P. Ling, USA

**1530. H. Polachek and R. J. Seeger, On shock-wave phenomena: Interaction of shock waves in gases,** Proc. Symposia appl. Math. 1, 119–144. American Mathematical Society, New York (1949).

The present paper constitutes a detailed summary of the existing mathematical theory concerning the interaction of plane shock waves in gases. The subject is treated economically but in detail and a brief review can do no more than suggest the contents. The Rankine-Hugoniot relations are first set up in a number of useful forms as the basis for the later material. Then the "simple" theory is exposed, i.e., the theory which assumes that the regions between the shocks, or between shocks and rigid boundaries, are regions of constant pressure. The cases of two-shock reflections ("regular" reflection) and three-shock (Mach or Y) reflections are fully treated. In the latter case a plane density discontinuity is involved. Recent experiments indicate a considerable discrepancy between theory and observation, and the authors develop a natural modification of the simple theory for both cases mentioned above. This consists in allowing a Prandtl-Meyer expansion to replace the constant pressure hypothesis in all regions where such an expansion wave is possible.

A third investigation is made of the refraction of shock waves at the intersection of two media of differing properties. One would expect a transmitted wave and a reflected wave. When the reflection is a shock, a general type of three-shock configuration results. The reflected wave may be a rarefaction wave, and a different configuration results. The complexity of the algebraic equations suggests a study of special cases to orient the investigator. The cases of normal incidence of a shock, and arbitrary incidence of an acoustic wave are accordingly studied. Special attention is given to the problems of an "impedance match," where the reflected wave is absent, and of total reflection. The final section contains some general results on multishock configurations. Tables and graphs are given wherever appropriate, and an excellent bibliography is appended. D. P. Ling, USA

**1531. Guy R. Bradshaw and Eugene B. Laskin, Experimental study of effect of vaneless-diffuser diameter on diffuser performance,** Nat. adv. Comm. Aero. tech. Note no. 1713, 28 pp. (Oct. 1948).

Four vaneless-diffuser diameters of 34, 27, 24, and 20 in. were studied by successively cutting down a 34-in. vaneless diffuser and running it in a variable-component rig with a commercial mixed-flow impeller to determine the variations in both compressor and diffuser efficiency with diffuser diameter. Runs were made at actual impeller tip speeds of 500 to 1300 ft per sec, through a range of air flows from open throttle to surge, to investigate the relation between diffuser efficiency and diffuser diameter under widely varying operating conditions. Total-pressure surveys were made at several radii throughout the diffuser and these, together with the standard measurements, provided information concerning the magnitude and the location of the important losses in efficiency through the diffuser.

Substantial variations in over-all compressor performance were produced by changes in diffuser diameter; over-all compressor efficiency increased with increasing diffuser diameter. In all the diffusers, losses in efficiency were limited to the entrance and exit regions, losses in the interior of the diffusers being negligible. Diffuser-exit losses increased from 3 to 15 points as diffuser diameter decreased, the losses being approximately inversely proportional to the square of the diffuser diameter. Diffuser-entrance losses amounted to about 0.06 and showed no systematic variation with diffuser diameter. In general, mean entrance and exit losses varied only slightly with load coefficient and tip speed.

No important effects of diffuser diameter on impeller efficiency were observed. Authors' summary

**1532. Leroy V. Humble, W. H. Lowdermilk, and M. D. Grele, Heat-transfer coefficients and friction factors for air flowing in a tube at high surface temperatures,** Heat Transf. fluid Mech. Inst., 165–180 (May 1949).

Tests were conducted on air in a 24 in., 0.402-in. ID inconel tube at Reynolds numbers up to 500,000 and average surface temperatures up to 1600 F. Satisfactory correlation between Nusselt number, friction factor, and Reynolds number was obtained by using physical properties of the air including density evaluated at the surface temperatures, and a velocity evaluated at the average bulk temperature for the conventional mass flow per unit cross-sectional area. N. A. Hall, USA

**1533. Michael F. Valerino, Generalized charts for determination of pressure drop of a high-speed compressible fluid in heat-exchanger passages: I—Air heated in smooth passages of constant area with constant wall temperature,** Nat. adv. Comm. Aero. res. Memo. no. E8G23, 51 pp. (Oct. 1948); J. aero. Sci. 16, 311–315 (May 1949).

In the first paper an analysis is made of the compressible-flow variations occurring in heat exchangers. Working charts are presented that permit convenient and accurate determination, at high subsonic Mach numbers and high rates of heating, of the pressure drop of air flowing in turbulent motion through smooth constant-area passages and heated at constant passage-wall temperature.

A method is given of incorporating the effect on the flow of high temperature differential between wall and fluid as based on recent NACA data. Good agreement is obtained between the experimental and chart pressure drops for wall temperatures as high as 1752 R (experimental limit) and for Mach numbers up to choke. Author's summary

The second paper outlines the theoretical derivation of the relations represented by the charts of the first one, and explains how the charts are to be used. The scope of the problem considered is restricted to turbulent flow of air in passages of uniform cross section, long enough to make entrance effects negligible, and having uniform surface temperature. R. Hosmer Norris, USA

**1534. W. R. Keagy and A. E. Weller, A study of freely expanding inhomogeneous jets,** Heat Transf. fluid Mech. Inst., 89–98 (May 1949).

The paper reports an experimental study of the effect of inhomogeneous density fields on mean concentration in freely expanding

jets. The effects of jets of helium, nitrogen, and carbon dioxide issuing from a sharp-edged orifice into still air were studied. It was found that the cross-sectional profiles of mean concentration and mean velocity in inhomogeneous gas jets exhibit the same type of similarity as do those of homogeneous gas jets and may be closely approximated by appropriate error functions. With such an approximation, expressions for the mean concentration and mean velocity along the axis of the jet may be obtained. As in tests by other experimenters, an apparent material and momentum loss was noted.

Lester L. Cronvich, USA

1535. **Rudolf Hermann, Theoretical calculation of the diffuser efficiency of supersonic wind tunnels with free jet test section,** Heat Transf. fluid Mech. Inst., 255–270 (May 1949).

One-dimensional analysis of the flow in tunnels with the test-chamber pressure equal to that in the nozzle exit is presented. The ratio of nozzle-exit area to diffuser-intake area is found to be an important parameter. The conditions at the end of the transformation zone in the diffuser inlet are determined. The general solution is found not to be a simple combination of transverse and oblique shocks with addition of other losses. The total losses are greater than those of a transverse shock. The velocity at the end of the transformation zone is subsonic. The solution contains the transverse shock as a special case (obtained when the above named area ratio equals unity). Diffuser efficiency is defined as the ratio of stagnation pressure after and before the diffuser divided by the ratio of stagnation pressure after and before a transverse shock at the test-section velocity. The diffuser throat area and the volume flow into the blowers after the tunnel are determined.

The paper terminates with a comprehensive comparison of the theory with experimental data from a number of European wind tunnels. The agreement is found to be good.

N. Holm Johannesen, Denmark

1536. **Robert Legendre, Blades profiles for slightly compressible flow** (in French), Bull. Ass. tech. Marit. Aéro. 46, 683–70 (1947).

In a previous paper (Bull. Ass. tech. Marit. Aéro., 1939), the author proposed a method for airfoil lattice tracing in incompressible flow; it was based on the study of the complex plane of the inverse of the complex velocity $V$. In this plane, a single closed contour corresponds with the airfoil lattice and a system of lines originating in two spiral vortexes with the stream lines. The author proposed to choose, in the $(1/V)$-plane, a particularly simple contour (e.g., two segments of straight lines passing through the origin $O$ and two circular arcs of center $O$), in order to impose on the contour of an airfoil in the physical plane a particular distribution of velocities corresponding to suitable pressure gradients.

In the present work, the method is extended to compressible flows. After a change of variable on the modulus of $V$, in order to obtain orthogonality of the lines of flow and equipotential lines in the hodograph plane, the approximate equation of state of Chaplygin-von Kármán-Tsien is assumed. It is then possible to introduce a new complex potential, an analytic function of the transformed velocity $\eta = f(\log \bmod V) + i \arg V$, and to reduce the problem to one for incompressible flow.

All the formulas necessary to the somewhat laborious development are derived, and mention is made of some criteria for the choice of the contour in the hodograph plane. Finally, two airfoil lattices are presented, both calculated by the author's method, one for incompressible, the other for compressible flow.

Gino Moretti, Argentina

1537. **Harvard Lomax and Max A. Heaslet, Linearized lifting-surface theory for swept-back wings with slender plan forms,** Nat. adv. Comm. Aero. tech. Note 1992, 41 pp. (1949).

This paper presents a generalized treatment of the transverse-flow, virtual-mass analysis, such as first employed by Munk and more recently by Jones. By means of this analysis the lift, and drag due to lift, of slender highly-swept wings and wing-body combinations at subsonic, transonic, and supersonic speeds may be predicted. For such bodies the streamwise velocity gradients can be neglected relative to those in the transverse plane, and the linearized potential equation reduces to Laplace's equation in $Y$ and $Z$.

This type of analysis is very useful and is now being widely used. The present paper, while treating no new cases, describes means of solving Laplace's equation not previously used in aerodynamics. This may make possible new solutions, but makes hard reading for the average engineer.

C. B. Smith, USA

1538. **M. Z. Krzywoblocki, On the boundary layer at a plane or tube in a periodically oscillating stream of compressible viscous fluid** (in English), Öst. Ingen.-Arch. 3, no. 5, 405–421 (1949).

This paper is an extension of earlier theoretical work in fluid dynamics, particularly that of Stokes, Rayleigh and Schlichting. Two aspects of the problem are considered, both of which are two-dimensional in character. The first is that of a plane surface parallel to the principal flow direction, while the second involves an oscillating stream flowing through a circular tube. In both cases the effects of viscosity and compressibility are taken into account, the former by means of the classical concept of the boundary layer.

The analysis is executed by setting up the two equations of motion, the equation of continuity, the equation of state for a perfect gas, and the equation of energy. The orders of magnitude of the various terms are then developed as in the case of Prandtl's classical treatment of the boundary-layer problem. A stream function is introduced as in Blasius' solution for the steady, incompressible flow past a flat plate.

Results are obtained for the longitudinal and transverse velocity components, the density and temperature variations across the boundary layer. While the analysis is outlined in some detail, only a limited amount of numerical calculations are given to indicate the significance of the results obtained.

M. J. Thompson, USA

1539. **I. Opatowski, Two-dimensional compressible flows,** Proc. Symposia appl. Math. 1, 87–93. American Mathematical Society, New York (1949).

In the first part of the paper the author shows the existence of several classes of steady two-dimensional flows of a compressible gas. These flows are actually degenerate three-dimensional flows. The principal tools used in the investigation are (1) a classification by T. Levi-Civita [Mem. Accad. Sci. Torino (2) 49, 105–152 (1899)] of degenerate metric tensors, and (2) a classification by the author of degenerate velocity-vector fields in which the components obtained by projecting the velocity vector on the tangents to the various coordinate lines are degenerate. By an examination of the potential equation for irrotational flows, the author shows that if the velocity components are degenerate in the above sense, then cylindrical, axial symmetric, helical, conical and spiral flows are possible. The details of the calculations are omitted. Further, the author claims that his calculations show that the above types of flow are permissible rotational flows.

In the second part of the paper, the author examines the prob-

lem of determining the two independent integrals for the stream function when suitable degeneracies exist in the contravariant components of the velocity. By recognizing the fact that $\rho a^{1/2}$ ($\rho$ density, $a$ local sound speed) constitutes a last multiplier of Jacobi, and, by use of the classical Jacobi theory, the author obtains formulas for the two desired independent integrals in the cases of spherical, helical, conical, and spiral flows.

It is not clear that, in the helical flows, the author is referring to the projected components of the velocity vector. There does remain the problem of examining degeneracies in the components of the velocity vector obtained by decomposing according to the parallelogram law. These latter components are as representative of the velocity vector as the projected components. That is, one set of components corresponds to use of covariant base vectors, the other (projected) set of components corresponds to use of the reciprocal contravariant base vectors.

*Courtesy of Mathematical Reviews* N. Coburn, USA

**1540. F. Wecken, Boundary positions of bifurcated compression shocks** (in German), Z. angew. Math. Mech. 29, 147–155 (May 1949).

The author is concerned with stationary triple-shock intersections. All the solutions possible according to the shock-wave equations are given by points on an algebraic surface in a $M_1^{-1}$, $\xi_a$, $\xi_b$ space, where $M_1$ is the upstream Mach number and $\xi_a$, $\xi_b$ are pressure ratios across the two consecutive shocks. He discusses the limiting cases $\xi_a = 1$, $\xi_b = 1$, $\xi_a = 0$, and $\xi_b = 0$ at some length, and also the eight "corner points" which represent doubly degenerate limiting cases. Some earlier papers are extended and corrected. W. R. Sears, USA

**1541. Ch. Roumieu, Study of two-dimensional unsteady flow in supersonic aerodynamics. Theoretical note on the transonic region** (in French), Rech. aéro. Paris, 47–54 (May-June 1949).

The article refers to two-dimensional supersonic nonstationary irrotational motion assuming that the potential $\Phi$ satisfies the linearized equation:

$$\Phi_{xx} + \Phi_{yy} - \Phi_{tt} = 0 \qquad (1)$$

($x$ and $y$ coordinates fixed in the air; sound velocity equal to unity). A profile is considered, never deviating much from a segment of the $x$-axis having unit length, its leading edge at $x = -f(t)$ and trailing edge $x = 1 - f(t)$. The motion is uniform or zero for $t < t_0$; a disturbance of the motion or a deformation of the profile starts at $t = t_0$, and $\Phi$ is calculated for $t > t_0$ with initial values $\Phi = \Phi_t = 0$ for $t < t_0$; boundary condition $\Phi_y = w_1(x, t)$ for $y = +0$, and $= w_2(x, t)$ for $y = -0$, with $w_1$ and $w_2$ determined by the form and the motion of the profile, and differing from zero only in a domain of the plane $xOt$ defined by $-f(t) < x < 1 - f(t)$.

The important problem is not the calculation of $\Phi$ in the whole of space, but the determination of $\varphi_1(x, t) = \Phi(x, +0, t)$ and $\varphi_2(x, t) = \Phi(x, -0, t)$ in that part of the plane $xOt$ which is influenced by the motion of the profile. The function $\varphi_1$ is obtained from the integral

$$\varphi_1(X, T) = -\frac{1}{\pi} \iint_h \frac{w_1(x, t)\, dx\, dt}{\sqrt{(T - t)^2 - (X - x)^2}} \qquad (2)$$

(there is a similar expression for $\varphi_2$), where $h$ is a part of the plane $xOt$ cut out by certain lines emanating from the point $X$, $T$. Equation (2) is proved by an application of Gauss' theorem to the solutions of equation (1), similar to the integral applied in Riemann's integration of the ordinary equation of sound. The pressure in a point of the profile can be found as $-\rho\partial\varphi/\partial T$; from this the lift and moment coefficients are obtained.

Three cases are considered: (1) Plane profile with sudden change of incidence: $t_0 = 0$, $f(t) = Mt$ (with $M > 1$); zero incidence for $t < 0$; $w_1 = w_2 = \alpha M$ for $t > 0$ ($\alpha$ small). (2) Arbitrary deformation of the profile, again with $f(t) = Mt$; as an example is taken $w_1 = w_2 = m(t) + \xi\omega(t)$, where $\xi = x + Mt$. (3) Plane profile in uniformly decelerated motion coming down to sound velocity at $t = 0$; velocity $1 - Nt$ for $t < 0$ ($N$ constant); $w = \alpha(1 - Nt)$, where $\alpha$ is the incidence angle. The order of magnitude of the pressure coefficient is calculated for infinitely small values of $N$. J. M. Burgers, Holland

## Turbulence, Boundary Layer, etc.

(See also Revs. 1498, 1499, 1500, 1513, 1515, 1565)

**1542. Benjamin Miller: Exploding a heat transfer myth,** Nucl. En. Power Aircr. Heat Transf. Lect. 1, 93–120 (Dec. 1948); **The laminar-film hypothesis,** Trans. Amer. Soc. mech. Engrs. 71, no. 4, 357–368 (May 1949).

In these papers (essentially identical) the author claims that up to the present time there has been no basis for belief in the existence of a "laminar sublayer" adjacent to the smooth solid boundary surface of a wall-type turbulent shear flow. He carefully traces the history of the sublayer idea, recalling that the famous experimental data of Nikuradse were adjusted to suit the hypothesis of a laminar sublayer.

Concerning experimental data, the reviewer would like to call attention to the velocity distributions of van der Hegge Zijnen (Figs. 39 and 40 of Ref. 7) and Reichardt (Fig. 1 of Ref. 13). These are Refs. 19 and 21, respectively, in the other version. These sets of data, showing undeniable derivations from the fully turbulent profile near the wall, have not been explained away by the author.

The author fails to mention the strong physical reasoning behind the assumption of a laminar sublayer, e.g., the certain requirement of zero fluctuation at the boundary plus the apparent existence of a "lower critical Reynolds number" for all laminar shear flows.

Furthermore, he does not emphasize the fact that the synthetic sharp partition of the flow field into "laminar," "buffer," and "turbulent" regions is merely intended as a crude artifice for approximate calculations, and is not taken literally by serious researchers in turbulence. It is unlikely that anyone working in turbulence research believes that there is any measurable region of literally zero fluctuation level near a solid surface.

Finally, the author neglects to mention the basic fact that there exists no genuine analogy between heat and momentum transfer in a turbulent flow; one is a scalar quantity and one is a vector, and they are described by nonidentical differential equations.

Stanley Corrsin, USA

**1543. J. Ginzel, A Pohlhausen-method for calculation of compressible laminar boundary layers** (in German), Z. angew. Math. Mech. 29, 6–8 (Jan.-Feb. 1949).

Besides the familiar momentum relation, the author employs the analogous energy-integral relation. He defines the usual displacement and momentum thicknesses, and also an enthalpy-loss thickness and a friction-loss thickness. After some manipulation, he arrives at a pair of coupled differential equations, and employs a Pohlhausen-type approximation based on polynomial profiles. The two equations are solved simultaneously by a step-by-step process. A few results are given, involving two different laws relating viscosity to temperature. It is mentioned that the heat transfer depends strongly on the ratio of temperature and velocity layer thicknesses. Unfortunately, this is only a concise summary

of the author's work, in which details of the method are not shown. It is not stated whether or where a complete paper is to appear.

W. R. Sears, USA

1544. **M. E. Shvets, On the approximate solution of some problems of hydrodynamics of the boundary layer** (in Russian), Prikl. Mat. Mekh. 13, no. 3, 257–266 (1949).

The author considers approximate solutions of some problems related to the boundary layer. These include: heat transfer in a laminar flow along a flat plate; cooling of a heated sphere in a laminar flow at small Reynolds number; free convection from a heated vertical plate; steady diffusion in a turbulent motion; and some other problems. The method used is a combination of the methods used in the boundary-layer theory and the method of successive approximations. A comparison of the approximate solutions of the above-mentioned problems with their exact solutions shows the effectiveness of the considered approximate methods.

E. Leimanis, Canada

1545. **L. G. Loĭtsyanskiĭ, Approximate method of integration of the equations of the laminar boundary layer in an incompressible gas** (in Russian), Prikl. Mat. Mekh. 13, 513–524 (1949).

The method is based on an application of successive momentum equations to the assumed family of velocity profiles in the boundary layer. As is well known, the main difficulty lies in the choice of the family of velocity profiles and the parameters of the family. The quantities

$$f = U'\delta^{**2}/\nu,\ \zeta = \tau_w\delta^{**}/\mu U,\ H = \delta^*/\delta^{**},$$

where

$$\tau_w = \mu(\partial u/\partial y)_{y=0},\ \delta^* = {}_0\int^{\delta}(1 - u/U)dy,$$
$$\delta^{**} = {}_0\int^{\delta} u/U(1 - u/U)dy,$$

are introduced as "form parameters" ($\delta$ can be infinite). Assuming the velocity profiles to be independent of $f$, $\xi$, $H$, i.e., assuming their similarity in various sections of the layer

$$u/U = \varphi(\eta);\ (\eta = y/\delta^{**}) \qquad (1)$$

the equations for $f$, $\zeta$, and $H$ are set up.

The simplest profile of velocities in the theory of asymptotic boundary layer is that for the flow along a flat plate placed edgewise in the fluid stream. In this case $\varphi(\eta)$ can be calculated from the tabulated values of the ratio $u/U$ as a function of $\xi = [y(U/\nu x)^{1/2}]/2$ [cf. Töpfer, Z. Math. Physik, 60, p. 397 (1912)]. This allows one to get a sufficiently precise solution of the boundary-layer equations for an arbitrary distribution of velocity $U$ at the free-stream boundary of the layer:

$$H = 2.59 - 7.55f,\ \zeta = 0.22 + 1.85f - 7.55f^2,$$
$$f = aU'/U^b\,{}_0\int^x U^{b-1}(\xi)d\xi = 0.44U'U^{-5.5}\,{}_0\int^x U^{4.5}(\xi)d\xi \qquad (2)$$

The results are found to agree well with the results of the first approximation given in a previous paper of the author [Doklady Akad. Nauk SSSR 35, 227–232 (1942)] and in a joint paper of Kochin and the author (Doklady Akad. Nauk SSSR 36, 262–266 (1942)). There is considerable divergence between the formulas of Wright and Bayley [J. aero. Sci. 6 (1939)] and the more precise formulas (2). This may be seen from the formulas

$$H = 2.59,\quad \zeta = 0.22 + 4.09f,\quad f = 0.44U'x/U,$$

expressing the results of Wright and Bayley in terms of the parameters of the author.

Finally the author shows how the method can be made more precise by replacing the velocity profiles (1) by either of a one-parameter family of velocity profiles $u/U = \varphi(\eta, f)$ or a three-parameter family $u/U = \varphi(\eta, f, \xi, H)$, the last case being not of great practical interest.

E. Leimanis, Canada

1546. **D. R. Hartree, A solution of the laminar boundary-layer equation for retarded flow,** Rep. Memo. aero. Res. Counc. Lond. no. 2426, 27 pp. (1949).

The laminar boundary-layer equation, for a linearly retarded velocity in the main stream, $U = 1 - \frac{1}{8}x$, in reduced variables, is solved numerically by working in finite intervals in $x$, with a correction for the finite length of $x$-interval. This work, carried out before the war, was performed to permit comparison of different methods of obtaining approximate solutions of the boundary-layer equation. Integrations were carried out numerically (to four decimals) for greater nominal accuracy. Separation is found close to $x = 0.9589$ in close agreement with Howarth who estimated it as 0.960, while von Kármán and Milliken's solution gives 0.816, and Pohlhausen's method gives separation at 1.248 for the linearly retarded velocity case.

The method of solving the partial differential equation for the boundary layer is to replace the derivatives with respect to $x$ by finite-difference ratios and to integrate along successive normals to the boundary at finite intervals, making use of the known velocity distribution at the start of the interval. For each interval a third-order nonlinear equation is obtained with two boundary conditions at $y = 0$ and one at $y \to \infty$. A trial-and-error form of solution is performed, starting with different values of the velocity gradient at the surface until one is found for which the condition at $y = \infty$ is satisfied. Two approximate forms of the boundary-layer equation are developed and used, one for use near the forward stagnation point and the other for greater distances. At $x = 0.4$ the results obtained with the first method agreed very satisfactorily with Howarth. The separation point was found by integrating from $x = 0.94$ to the point at which the wall velocity gradient is zero, and secondly, by carrying the integrations up to $x = 0.958$ and empirically extrapolating. Taking advantage of Goldstein's analysis [Quart. J. Mech. appl. Math. 1, p. 43 (1948)] of the flow upstream from separation, the author finds the same separation point to within the fourth decimal. This approach also yields an expression for the velocity profile at separation in essential agreement with the results of the integration process. Consideration is given to the singularity at the separation point.

J. M. Robertson, USA

1547. **H. Schlichting, An approximate method for calculation of the laminar boundary layer with suction** (in German), Ing.-Arch. 16, 201–220 (1948).

The approximate calculation of the flow in an incompressible laminar boundary layer with boundary-layer suction is undertaken, using the integral form of the momentum equation and choosing an appropriate one-parameter family of velocity profiles. The distribution of velocity $u$ parallel to the surface is taken, when the free-stream velocity is $U$, to be $u/U = F_1(\eta) + KF_2(\eta)$, where $\eta = y/\delta_1(x)$ and $K$ depends upon $x$ for a given problem. Here $F_1(\eta)$ is chosen to be the asymptotic solution for uniform boundary-layer suction: $F_1(\eta) = 1 - e^{-\eta}$ [Pretsch, Z. angew. Math. Mech. 24, 264–267 (1944)], while $F_2(\eta)$ is chosen to give a good approximation to the Blasius profile for the flat plate without pressure gradient:

$$F_2(\eta) = \begin{cases} F_1(\eta) - \sin\left(\frac{1}{6}\pi\eta\right),\ 0 < \eta < 3, \\ F_1(\eta) - 1 = -e^{-\eta},\ \eta \geqq 3. \end{cases}$$

The value of $K(x)$ is completely determined by the boundary conditions and the distribution of suction velocity at the surface. Consequently all features of the solution follow from the deter-

mination of the one parameter $Z = \vartheta^2/\nu$, where $\vartheta$ is the momentum thickness. The differential equation for $Z$ which results from the momentum integral equation may be solved by the isocline method for an arbitrary distribution of free-stream and suction velocities. Charts of the parameters involved are provided to assist in the numerical process which is completely described.

Several examples illustrate the application and accuracy of the method. The examples of the flat plate, circular cylinder, and symmetric Joukowsky profile at zero angle of attack, all with uniform suction, are presented in detail.

*Courtesy of Mathematical Reviews* F. E. Marble, USA

1548. **Lester Lees, The stability of the laminar boundary layer in a compressible fluid,** Nat. adv. Comm. Aero. Rep. no. 876, 47 pp. (1947).

See Rev. 1, **485**.

1549. **Raymond M. Comolet, On perturbations of laminar flow** (in French), C. R. Acad. Sci. Paris **229**, no. 6, 415–417 (1949).

This paper describes observations of the critical Reynolds number for the transition from laminar to turbulent flow in a cylindrical tube. The flow was disturbed by the injection of liquid with mean velocity $v$ through a hole of diameter $h$ in the wall of the tube. The critical Reynolds number $VD/\gamma$ was found to be a function of $vD/Vd$ ($V$ mean velocity of flow in tube, $D$ diameter of tube). No explanation is offered.

G. K. Batchelor, England

1550. **H. H. Pearcey, Profile drag measurements at compressibility speeds on aerofoils with and without spanwise wires or grooves,** Rep. Memo. aero. Res. Counc. Lond. no. 2252, 21 pp. (1949).

Tests on NACA 2218 and EC 1250 sections of 5-in. chord over a Reynolds number range from $10^6$ to $1.8 \times 10^6$ showed that wires of 0.006-in. diam are unsuitable for fixing transition due to compressibility and drag effects of the wires themselves.

Further tests on an EC 1250 section of 12-in. chord over a Reynolds number range from $10^6$ to $4.4 \times 10^6$ show that the same wires were successful in fixing transition. These tests confirmed theoretical predictions that there is practically no compressibility increase in profile drag below the critical Mach number.

Additional tests on NACA 2417 section of 2-in. chord showed triangular grooves to be ineffective in fixing transition.

H. P. Liepman, USA

1551. **Laurence K. Loftin, Jr. and Dale L. Burrows, Investigations relating to the extension of laminar flow by means of boundary-layer suction through slots,** Nat. adv. Comm. Aero. tech. Note no. 1961, 58 pp. (1949).

These tests were made in a tunnel having a stream-turbulence level up to 0.2%. All measurements were made with pressure tubes and orifice plates. Several types of slots were tried, as well as perforated plates and screens; slot spacing was also varied. Slots in both favorable and unfavorable pressure gradients were investigated.

It was found that the requirements of minimum pressure loss and optimum effect on boundary layer are not completely compatible. Nevertheless, tests on an NACA 0007-34 profile showed considerable success in increasing the extent of laminar boundary-layer flow, with modest expenditures of suction power, for airfoil Reynolds numbers up to about $7 \times 10^6$. Above this, it seemed impossible to avoid a change of boundary-layer profile toward the characteristic turbulent shape, behind the slots, although boundary-layer thickness did not increase greatly. It is suggested that laminar-boundary-layer oscillations were present. The drag-coefficient equivalent of the suction power used for best extension of laminar flow at $7.5 \times 10^6$ was 0.00031 for one surface.

Based on the results of these tests, an analysis has been made of slot power losses. This is intended to permit a rational design of slots for any configuration. W. R. Sears, USA

1552. **L. Onsager, Statistical hydrodynamics,** Nuovo Cim. Supplement 6, no. 2, 279–287 (1949).

The author first considers statistically the motion of $n$ parallel vortexes in an incompressible frictionless fluid of finite volume, and shows that vortexes of the same sign tend to approach one another if the total energy is greater than a certain value. Eventually, the large compound vortexes formed in this way remain as the only conspicuous features of the motion. The relevance of this idealized model to the real problems with continuous distributions of vorticity, and the possibility of three-dimensional motion are discussed. The characteristics of fully turbulent motion are then reviewed with especial reference to the concept of local similarity. Redistribution of energy among the infinite number of possible degrees of freedom is visualized as an accelerated cascade process, in which the first few steps determine the rate of energy dissipation. Then the energy density in the later stages of the process will be determined by the rate at which energy is being handed down, and dimensional arguments may be used to find forms for the energy distribution function and the scale-dependent diffusion coefficient. Observations of atmospheric diffusion have confirmed this theoretical form. Finally, it is pointed out that, in principle, dissipation of energy can occur in the absence of viscosity, since the ordinary proof of the conservation of energy is not valid then. A. A. Townsend, England

1553. **C. F. von Weizsäcker, The turbulence spectrum at large Reynolds' numbers** (in German), Z. Phys. **124**, 614–627 (1948).

The author considers a volume filled with isotropic turbulence of linear dimension $L_0$, subdivided into successively smaller volumes of linear dimensions $L_n$, and discusses the mean value of the physical quantities over these volumes. The condition of equilibrium of energy dissipation leads to the following law: the average value of the magnitude of relative velocity for two parts at a distance $L_n$ apart is proportional to $L_n^{1/3}$. The turbulence exchange over a distance $L_n$ is then proportional to $L_n^{4/3}$. In terms of the Fourier analysis of the flow field, this means that the spectrum is proportional to the negative five-thirds power of the wave number. C. C. Lin, USA

1554. **Howard W. Emmons, The numerical solution of the turbulence problem,** Proc. Symp. appl. Math. I, 68–71, (1949).

The author suggests that the development in time of a field of turbulent motion might be computed directly from finite-difference representations of the equations of continuity and of momentum. The initial distribution of velocity could be taken as the experimental mean velocity with a superimposed random variation. Presumably, after sufficient time, the solution would be such that averages (of velocity, mean-square velocity, etc.) over an appropriate space coordinate (e.g., parallel to the length of a pipe) vary with time in a manner which is independent of the assumed initial conditions.

The suggestion is tried out on the case of two-dimensional flow in a channel, the flow field being taken as periodic in the flow direction with wave length equal to 2.4 channel widths. The calculation is carried out for one time interval only, which is not sufficient to smooth out the initial kinks in the distribution of averaged

quantities so that the results are of limited value. The author recommends the use of large-scale computing machines.

G. K. Batchelor, England

1555. **E. R. G. Eckert, Interferometric studies of beginning turbulence in free and forced convection boundary layers on a heated plate,** Heat Transf. fluid Mech. Inst., 181–190 (May 1949).

The Zehnder-Mach interferometer was used to study boundary-layer transition. Free-convection studies were made mostly in the upstream part of the transition region. The critical oscillations were found to have a wave length 3.1 times the boundary-layer thickness and a velocity 0.73 times the maximum. Temperature profiles are definitely laminar for Grashof numbers less than about $10^9$. A plot of the film-heat-transfer coefficient versus distance shows agreement with laminar free-convection theory at low Grashof numbers and at higher values a scatter around a value agreeing with King.

Forced-convection flow studies were made in a small air tunnel. The critical oscillations had a wave length about 13 times the displacement thickness. Laminar temperature profiles were found up to Reynolds numbers of about 100,000. Comparison of several temperature profiles at a Reynolds number of 300,000 on a universal logarithmic plot shows good agreement with predictions of the analogy between heat and momentum transfer. It appears that in the region outside the buffer layer the turbulent diffusivity for heat is about 1.4 times that for momentum. Measured film-heat-transfer coefficients again show a scatter between the accepted laminar and turbulent relations due to flow oscillations.

I. M. Robertson, USA

1556. **J. G. Oldroyd, A suggested method of detecting wall-effects in turbulent flow through tubes,** Proc. int. rheolog. Congr. Holland, II: 130–134; III: 44–46 (1948).

Recent experiments on the turbulent flow of high-polymer solutions through tubes have shown some unexpectedly high rates of flow which, it is suggested, may be caused by abnormal behavior of the fluid in the thin laminar sublayer at the tube wall. Solutions which show this abnormal behavior are essentially non-Newtonian viscous liquids. In the present paper the arguments hitherto applied to the turbulent flow of Newtonian liquids are extended to the turbulent flow of non-Newtonian liquids. The result is the equation

$$y = B \log_{10} x - (B \log_{10} \eta_1 + C) \qquad (1)$$

where $B$ and $C$ are constants,

$$x = \eta_1 R_1 \gamma^{1/2} = 2a(-a\rho\, dp/dz)^{1/2}$$
$$y = \gamma^{-1/2} = (-a\rho^{-1} dp/dz)^{-1/2} Q/\pi a^2$$

$\eta_1$ is the limiting value of the coefficient of viscosity for high rates of shear, $R_1$ the Reynolds number $2\rho Q/\pi a \eta_1$, $\gamma$ the wall-friction coefficient, $a$ the radius of the tube, $\rho$ the fluid density, $dp/dz$ the axial pressure gradient, and $Q$ the volume rate of flow.

When an effective velocity of slip at the tube wall is present, $Q$ is replaced by $Q - \pi a^2 w$ where $w$ is the velocity of slip. Equation (1) then becomes

$$y = B \log_{10} x - (B \log_{10} \eta_1 + C) + \zeta x/48$$

where $\zeta = -2w/(a\, dp/dz)$.

To detect a wall effect, it is suggested that $y$ be plotted against $\log_{10} x$: both $y$ and $x$ are observable functions of the pressure gradient, of the volume rate of flow, and of the tube dimensions. If, at sufficiently large Reynolds numbers, all the results for different tubes lie on a single curve, which should be a straight line, the fluid is assumed to behave normally. If, however, different curves result for different tube diameters, a wall effect is indicated.

It is stated that the suggested method for detecting wall effect is to be regarded as a tentative one, to be judged by the success of the method in practice.

Neal Tetervin, USA

## Aerodynamics of Flight; Wind Forces

(**See also Revs. 1515, 1550, 1615**)

1557. **Philip Donely, Summary of information relating to gust loads on airplanes,** Nat. adv. Comm. Aero. tech. Note. no. 1976, 145 pp. (1949).

The report correlates and coordinates the available information through October 1947 on gust structure, airplane reaction, and pertinent operating statistics with reference to the prediction of gust loads on airplanes. The material is based mainly on the past work of the NACA, much of which has been accomplished through the cooperation of the several interested government agencies and the airlines.

The available information on gust loads does not permit the determination of applied loads under actual operating conditions on an absolute basis. The procedure has been to transfer load data from a reference airplane to new airplanes by using available knowledge of aircraft behavior. The important features and implications in this procedure are that the same procedures must be used to evaluate loads data as used in load calculations. In addition, the procedure presupposes no drastic changes in airplane configuration or in the airplane-operating conditions.

A brief discussion is given of gust-alleviating systems so as to reduce the loads and improve the riding qualities of high-speed aircraft, although these two requirements are not necessarily compatible.

The detailed information contained in the report will be valuable to all persons working with this problem. The report has three main sections which are entitled: The structure of atmospheric gusts, Airplane reactions, and Operating statistics.

Conrad A. Lau, USA

1558. **J. H. Greidanus and A. I. van de Vooren, Proposal for an airworthiness requirement referring to symmetrical gust loads,** Nat. Luchtlab. Amsterdam Rep. no. F45, 1–11 (1949).

This report presents a recommendation on gust-load requirements intended for the Dutch Airworthiness Standards. The proposal is discussed and compared with the existing recommendation.

Harvard Lomax, USA

1559. **Albert E. McPherson, J. Evans, Jr., and Samuel Levy, Influence of wing flexibility on force-time relation in shock strut following vertical landing impact,** Nat. adv. Comm. Aero. tech. Note no. 1995, 41 pp. (1949).

Tests were conducted to determine the force developed in a shock strut as a function of the flexibility of the attached wing structure. It was found that for a duration of impact $T_I$ greater than 1.5 times the natural period $T_N$ of the wing, the force-time relation in the shock strut was substantially the same as though the flexible structure had been replaced by a rigid body having the same net weight. The peak force for $1.5 < T_I/T_N < 2.5$ showed a reduction of up to 10%, and the peak acceleration at the center a reduction of up to 15% due to flexibility. These reductions were somewhat greater than the probable experimental error of about 5%.

An analysis of the effect of wing flexibility on the impact force was also carried out. It was found that for $0.231 < T_I/T_N < 2.47$ and for $1 < M_1/M_0 < \infty$, where $M_1$ is the generalized mass of the

wing in its fundamental mode and $M_0$ is its actual mass, the impact force for the flexible wing was 0.775 to 1.000 times that for the rigid wing. For current designs of large airplanes with $T_f/T_N \sim 1$ and $5 < M_1/M_0 < 15$, the impact force for the flexible wing would be about 0.95 times that for the rigid wing.

A formula, based on the analysis, is given for the ratio of impact force with a flexible wing to impact force with a rigid wing. This formula checks the experimental data within the experimental error. From authors' summary by R. L. Bisplinghoff, USA

**1560. Richard C. Dingeldein and Raymond F. Schaefer, Full-scale investigaion of aerodynamic characteristics of a typical single-rotor helicopter in forward flight,** Nat. adv. Comm. Aero. Rep. no. 905, 28 pp. (1948).

A series of charts, based on results of tests in the Langley full-scale tunnel, are given for the performance characteristics of a helicopter with a single main rotor and a tail rotor. These charts include plots of rotor lift coefficient and useful drag-lift ratio as functions of thrust coefficient $C_T$, helicopter angle of attack, power drag-lift ratio, and advance ratio $\mu$. Values of $\mu$ range from 0.10 to 0.27, while $C_T$ ranges from 0.0030 to 0.0060. Aerodynamic characteristics of the fuselage are also shown. These charts can be used for a rapid estimation of rotor forward-flight performance, provided that the actual rotors have physical properties similar to the test rotors, although the solidities need not be the same.

Tests were made with two types of rotors. One was of ordinary surface finish, and the other had unusually smooth surfaces. The results of the tests implied that the smooth blades required appreciably less horsepower than the ordinary blades (13% less power for flight at airspeeds of from 44 to 60 mph). Moreover, it was found that (a smaller) power saving could also be realized by operating at higher thrust coefficients.

Comparison of the results of this investigation with those of flight tests and of existing theory showed fair agreement.

Morris Morduchow, USA

## Aeroelasticity (Flutter, Divergence, etc.)

**1561. A. I. van de Vooren, Diagrams of flutter, divergence and aileron reversal speeds for wings of a certain standard type,** Nat. Luchtlab. Amsterdam Rap. no. V. 1397, 59–69 (1947).

**1562. A. I. van de Vooren, The treatment of a tab in flutter calculations including a complete account of aerodynamic coefficients,** Nat. Luchtlab. Amsterdam Rap. no. V. 1386, 117–142 (1947).

In this paper the equations of motion for a wing-aileron-tab system, expressing the equilibrium of inertia, elastic and aerodynamic forces, are deduced. Explicit formulas permitting the immediate computation of these forces from the constructional data of the system are also given. The elastic forces of the control mechanism, which appear to depend on the type of tab considered, are derived for all common tab designs (spring tab, trim tab, balance tab, etc.). Throughout the whole report compressibility effects are neglected.

From the author's summary by N. O. Myklestad, USA

**1563. A. S. Batson and J. H. Warsap, Note on the effect of deformation of a control flap on lift and hinge moment,** Rep. Memo. aero. Res. Counc. Lond. no. 2315, 7 pp. (1949).

Wind-tunnel tests have been conducted to determine the effects of deformations of the upper and lower surfaces of a trailing-edge flap on the lift and hinge moment characteristics of an airfoil with a 0.15c flap. The deformations were introduced systematically by making the upper and lower flap surfaces either concave or convex so that both the mean camber and trailing-edge angle of the flap could be varied independently of the other. It was found that the slopes of the hinge-moment coefficient with respect to angle of attack and flap angle were constant over limited ranges of these angles, and were functions of trailing-edge angle only (independent of flap mean camber). Conversely, the flap-hinge moment and airfoil lift at zero angle of attack and flap angle were found to be linear functions of flap camber, and were independent of trailing-edge angle.

The tests were conducted at a Reynolds number of approximately one million and at low Mach number, using a symmetric modified low-drag airfoil of 15% thickness ratio.

William F. Milliken, Jr., USA

## Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 1512, 1514, 1531, 1560)

**1564. H. Hausenblas, Temperature of turbine blades** (in German), Motortech. Z. 10, 9–10 (Jan.-Feb. 1949).

An analytical method is presented for determining the distribution of turbine-blade temperature along the length of a blade for a given distribution of gas temperature and given blade-base temperature. The solution is applicable for both solid and internally cooled blades. A parameter $\alpha U/\lambda F$ is assumed constant along the blade length ($\alpha$ coefficient of heat transfer between gas blade; $U$ perimeter of blade cross section; $\lambda$ coefficient of thermal conductivity for blade material; $F$ cross-sectional area of blade).

A. R. Bobrowsky, USA

**1565. C. K. Ferguson, Mixing of parallel flowing streams in a pressure gradient,** Heat Transf. fluid Mech. Inst., 77–88 (May 1949).

Although the jet pump in one form or another has been the subject of several recent investigations, no satisfactory theory exists for the pressure and velocity variations throughout the mixing chamber. On the basis of new average experimental measurements made with air at near atmospheric pressures in a jet pump of rectangular cross section and without diffusor (and other experimental results from Viktoria), a method to calculate the location and magnitude of the maximum pressure rise in the mixing chamber was developed. Since the initial mixing region of the jet occurs at nearly constant pressure, Kuethe's solution was applied to the jet-spreading analysis. The pressure rise with distance downstream from the constant pressure region was determined, following methods of Flugel. Reasonable correlation of theory and experiment was demonstrated.

Richard G. Folsom, USA

## Flow and Flight Test Techniques

(See also Revs. 1458, 1514, 1535)

**1566. A. S. Halliday and P. H. Allwork, The new whirling arm at the National Physical Laboratory,** Rep. Memo. aero. Res. Counc. Lond. no. 2286, 15 pp. (1949).

This report presents a detailed description of a new whirling arm which is used to measure the rotary derivatives $L_r$, $N_r$, and $M_q$ of airplane models. The radius of the model path and the maximum linear speed of the model are 30 ft and 100 fps, respectively.

A few remarks on the calibration of the working section (6 ft

9 in. high and 8 ft 9 in. wide) for wind speed and direction are included.

J. Buhrman, Holland

1567. **B. Thwaites, On the design of contractions for wind tunnels,** Rep. Memo. aero. Res. Counc. Lond. no. 2278, 9 pp. (1949).

The purpose of this paper is to develop a mathematical method of computing the shape of three-dimensional contractions suitable for wind-tunnel applications. One of the main requirements of the contraction, as considered by the author, is shortness combined with negative velocity gradients on the surface, small enough to prevent flow separation. The problem, therefore, is to compute a shape which has a specified velocity distribution along the surface. A general solution is obtained of three-dimensional axisymmetric incompressible inviscid irrotational flow taking place between two equipotential planes perpendicular to the axis. In the solution, the axial velocity is expressed as a Fourier cosine series, and the type of flow corresponds to that simulated in an electric tank. The author has been using this method to advantage in computing a variety of shapes of wind-tunnel contractions and testing them in the electric tank, with cylindrical ends fitted to the contraction in order to simulate the flow occurring in actual wind tunnels.

Frank L. Wattendorf, USA

1568. **S. A. Schaaf, D. O. Horning, and E. D. Kane, Design and initial operation of a low density supersonic wind tunnel,** Heat Transf. fluid Mech. Inst., 223–242 (May 1949).

A description of a low-density supersonic wind tunnel and its accessories is presented by the authors. This tunnel, built for research in superaerodynamics, can be described as a once-through, continuous flow, open-jet unit. It is driven by a 5-stage steam ejector, has a 4-in-diam nozzle operating at $M = 4.0$, with a Reynolds number range from 0.24 to 12000, and a static-pressure range from 20 to 200 microns Hg absolute. The paper contains also a description of the instrumentation and a discussion of the model laws for rarefied gases. An adaptation of the method of characteristics for the calculation of the isentropic core of the supersonic nozzle, and the method used for boundary-layer correction are outlined in an appendix.

Andrew Fejer, USA

1569. **L. M. Milne-Thomson, Applications of elliptic functions to wind tunnel interference,** Proc. roy. Soc. Edinburgh Sect. A. 62, part 3, 316–318 (1948).

The author considers a wind tunnel whose section is a given curve in the $z$-plane, the region interior being mapped within the unit circle of a $Z$ plane by $z = f(Z)$. A general formula is then written for the additional upwash (interference velocity) at a wing of elliptic loading, in terms of $f(Z)$. The particular cases of elliptical and rectangular tunnels are treated; for these, the transformation appears in suitable Jacobian elliptic functions. For the elliptic tunnel, the formulas are worked out in some detail, assuming the wing to lie on the major axis. Finally, when the wing tips coincide with the foci, the formulas can be integrated in closed form, giving results in agreement with Glauert's [British Aero. Res. Comm. Rep. and Memo. no. 1470 (1932)].

W. R. Sears, USA

1570. **D. P. Riabouchinsky, Hydraulic analogy of the motion and resistance of a compressible fluid as an aid to aeronautical research,** Reissner Anniv. Vol., 61–88, J. W. Edwards, Ann Arbor (1949).

This paper presents a useful and interesting review of the theoretical basis for the analogy between supersonic flow in a gas and surface wave motion in incompressible liquids, such as water. The equations of motion for the two cases, neglecting viscosity, are presented, and the various approximations generally introduced are discussed. Systems representing both relative and absolute motions are considered.

Such problems as the analogy between the normal shock and the hydraulic jump are discussed, as well as the use of a water basin with flowing water and stationary model and the towing-tank procedure. Both small and large-scale equipment are described.

A number of problems in supersonic flow are studied in detail by the experimental techniques with the addition of shadowgraph photography. These include the behavior of two-dimensional ducts without and with internal choking, Hartmann's acoustic generator, and airfoils with attached and detached shocks.

The effects of viscosity, surface tension, and other similar modifying effects are briefly considered.

M. J. Thompson, USA

1571. **Otto Schumacher, Pressure gages with plate springs** (in German), Arch. tech. Messen: no. 161, V1343–7 (May 1949); no. 163, V1343–8 (Aug. 1949).

The paper discusses the following topics in the title gages: range of pressures, method of mounting, precision of readings, form of the construction of the plate spring (section, material, fixing); empirical formulas for calculation; special forms of construction; and transmission mechanism to the reading or registering device.

Albert Schlag, Belgium

1572. **T. L. Chambré and H. R. Smith, The impact tube in a viscous compressible gas,** Heat Transf. fluid Mech. Inst. 271–278 (May 1949).

This paper interprets the measurements of impact pressure in subsonic flows at low pressure. A theory is presented showing that the viscous forces should be taken into account as well as the inertia forces, and the experimental data reported indicate that the former may be of the same order of magnitude as the latter for certain experimental conditions.

A theoretical expression is derived for the pressure measured by an impact tube in subsonic flow where the effects of compressibility and viscosity are taken into account. It is shown that the expression for the stagnation pressure $P_{st}$ is made up of two parts: (a) the usual inviscid term, $P_R$, and (b) a term due to the viscosity:

$$P_{st} = \{P_B + \tfrac{1}{2}\,\rho_B\,V_B^2\} + 2\,\mu\,\beta \qquad (1)$$

where

$$P_B + \tfrac{1}{2}\,\rho_B\,V_B^2 = P_R = P_1\left(1 + \frac{\gamma - 1}{2}\,M_1^2)\right)^{(\gamma/\gamma - 1)}$$

$\mu$ is the coefficient of viscosity, $\beta$ a constant with dimensions of a velocity gradient, and $P_1$ and $M_1$ are the free-stream pressure and Mach number. The authors deduce that the pressure recorded by the impact tube under viscous-flow conditions is always larger than the pressure recorded if viscosity effects are neglected.

By dividing equation (1) by the dynamic head, $\frac{1}{2}\rho_r V_r^2$, the viscosity term is transformed into one which is inversely dependent upon the Reynolds number of the flow. Thus it is shown that as the Reynolds number decreases the viscosity term becomes larger.

The constant $\beta$ is evaluated for a hemispherical tube of radius $b$ as

$$\beta = \frac{V_1}{2(b + \delta^*)}\left(\frac{29}{8} - \frac{31}{34}\,M^2\right) \qquad (2)$$

where $V_1$ is the free-stream velocity, $M$ is the Mach number, and

$\delta^*$ is the boundary-layer thickness added to $b$ to give the effective radius of curvature at the nose of the impact tube. $\delta^*$, in turn, is expressed as a function of Mach number and Reynolds number. In this manner equation (1) is finally expressed in terms of Mach number, Reynolds number, pressure, temperature, viscosity, ratio of specific heats of the gas, and radius of the impact tube.

To illustrate the application of equation (1) the results of measurements are given in tabular form for an impact tube placed in a subsonic stream at low densities, about 100 microns static pressure. The Mach number range was from 0.3 to 0.7, and it was shown that the theoretical values of volumetric rate of flow based on impact tube (of 0.075 in. radius) data gave within about 10% of the actual rates when the viscosity term was used, and varied from the actual rates by 70 to 150% when the viscosity term was neglected. F. K. Hill, USA

**1573. John A. Zalovcik, A radar method of calibrating airspeed installations on airplanes in maneuvers at high altitudes and at transonic and supersonic speeds,** Nat. adv. Comm. Aero. tech. Note no. 1979, 24 pp. (1949).

A method of calibrating the static-pressure source of a pitot-static airspeed installation on an airplane in level flight, dives, and other maneuvers at high altitude and at transonic and supersonic speeds is described. The method principally involves the use of radar-phototheodolite tracking equipment. The various sources of error in the method are discussed, and sample calibrations are included.

From author's summary by P. Donely, USA

**1574. Gian Battista Nicolò, Flight at speed near to the lower limit of the transonic field** (in Italian), Riv. aero. 25, no. 7, 425–432 (1949).

The author gives a short, but rather complete view of the difficulties in determining in advance the aerodynamic coefficients of an airplane at high subsonic speed. As analytic means are still rather poor, he describes the possibilities and difficulties of free-flight experiments. Results obtained with several modern planes are given in diagrams and discussed. I. Flügge-Lotz, USA

**1575. L. S. G. Kovasznay, Technique for the optical measurement of turbulence in high speed flow,** Heat Transf. fluid Mech. Inst. 211–222 (May 1949).

The author proposes to use the statistical properties of the light fluctuations on shadowgraphs of highly turbulent compressible flows in order to obtain information about the statistical structure of the so recorded turbulence. Since the relative variations in brightness on a shadowgraph are in the first approximation proportional to the integral in the light direction of the Laplacian of the density in a plane perpendicular to the latter, only very rapid fluctuations are detectably recorded; but it is exactly this high-frequency end of the turbulent spectrum which is inaccessible to the currently principal tool of experimental turbulence research, the hot-wire anemometer. Using the above linear relation between the flow density distribution and the relative brightness on the shadowgraph, and assuming that the amount of darkening of the final photographic record varies linearly with the relative brightness of illumination, it is shown that the two-dimensional spectrum of the shadowgraphic light distribution is the same as that of the Laplacian of the density in the median plane of the turbulent flow, provided the turbulence is homogeneous and the transluminated turbulent region sufficiently wide. The same is therefore true regarding the corresponding power spectra, which in their turn can be computed from the correlation function of the light distribution in the shadowgraph. No completed experimental reductions by this procedure are reported, but correlation functions have been determined for the turbulent wake of a 37-mm shell traveling at Mach number 1.8. The scale of turbulence is found to be an order of magnitude below the operating range of modern hot-wire anemometers.

F. J. Weyl, USA

**1576. Raymond M. Comolet, New method to determine the position of a meniscus. Application to manometers** (in French), C. R. Acad. Sci. Paris 229, no. 18, 867–868 (1949).

A method of indicating the position of a meniscus by measuring the electrical resistance of the capillary layer is described. A micromanometer using this method has been constructed by the author, with a gas bubble between electrodes at the closed end of a manometer tube containing a conducting liquid. Variations in pressure cause changes of volume of the gas bubble, which variations are measured by the change of resistance between the electrodes. The author states that this method permits the measurement of rapidly varying pressures and reduces inertia effects.

John A. Lewis, USA

## Thermodynamics

(See also Revs. 1479, 1531, 1578, 1588, 1590, 1610)

**1577. Georges Claude and André Claude, On the possibility of extracting heat from the earth** (in French), C. R. Acad. Sci. 228, no. 22, 1701–1702 (1949).

The possible use of the earth's heat through large-scale building of artesian wells is discussed. A calculation based on formulas previously published shows that wells built in sandy soil or limestone can have a high rate of discharge of water, whereas wells built in loamy soil would be useless because of low permeability. The economic feasibility of the scheme is not touched upon.

Herbert Riehl, USA

## Heat and Mass Transfer

(See also Revs. 1456, 1532, 1533, 1542, 1543, 1544, 1555, 1577)

**1578. Max Jakob and I. B. Fieldhouse, Cooling by forcing a fluid through a porous plate in contact with a hot gas stream,** Heat Transf. fluid Mech. Inst. 191–200 (May 1949).

The article deals with a study of the comparison of the heat transfer from a stream of hot gas to a porous plate through which cooling fluids were forced, with a solid plate cooled by a fluid. The results indicated that the heat transfer from a hot gas to a plate can be reduced by use of a porous plate through which a cooling fluid is forced. The experimental results obtained using hot-air streams at 400 and 700 F and velocities from 300 to 600 fps indicated that the coefficients of heat transfer were reduced from 63 to 73% by use of a porous plate compared to a solid plate. The data indicated that even greater effects may be expected at higher gas temperatures. The coefficient of heat transfer was found to be almost the same whether water, steam, or nitrogen left the exposed surface of the porous plate. Owing to differences in heat capacities and latent heats, less cooling water was needed if evaporated in the porous plate than without evaporation. More nitrogen than water was required to obtain the same cooling effect. G. A. Hawkins and F. E. Romie, USA

**1579. D. I. Boyarintsev, The heating of a cylinder by solar radiation** (in Russian), Izv. Akad. Nauk SSSR Ser. tekh. Nauk 1949, 1469–1480 (1949).

The author considers the heating of cylindrical tanks with hemispherical ends, and of a pipe line, due to direct and reflected

solar radiation. First, through spherical trigonometry, the rate $Q$ of heat gain is found. Then the heat-balance equation leads to a second-order differential equation for the temperature difference between the contents of the tank and the surrounding temperature. A series solution of this equation is given for the case when $Q$ is a function of time alone. Some numerical calculations are included.

*Courtesy of Mathematical Reviews* R. E. Gaskell, USA

1580. **J. C. Bell and E. F. Katz, A method for measuring surface heat transfer using cyclic temperature variations,** Heat Transf. fluid Mech. Inst. 243–254 (May 1949).

Ångstrom's method for determining thermal conductivities of solids has been developed by the authors to attack the measurement of surface heat transfer. A cyclic (preferably a sinusoidal) variation in temperature is imposed on the fluid stream just upstream from the surface. A combination of Wheatstone bridges, d-c amplifiers, and oscillograph is used to determine the change in amplitude and phase of the temperature cycle as the fluid stream passes the surface being investigated.

Although, as the authors state, much developmental work remains to be carried out, the method outlined offers great promise. In the past the difficulty has been in accurately determining the temperature of the wall for comparison with that of the passing fluid. The new method avoids the necessity of measuring the surface temperature.

The mathematical theory necessary for the interpretation of the oscillographic record is provided in the paper, together with the description of the apparatus used and the limited experimental results obtained to date. The Nusselt numbers determined experimentally by the new method are compared with values calculated from equations in general use. Correlation is not of a high order, but further work may possibly reduce the disagreement.

R. E. Button, Australia

1581. **L. K. McCune and R. H. Wilhelm, Mass and momentum transfer in solid-liquid system,** Indust. Engng. Chem. 41, no. 6, 1124–1134 (June 1949).

This paper deals with mass transfer between an upward stream of liquid and solid particles in consolidated and in expanded, fluidized beds. The solid particles in question were 2-naphthol and the liquid was water. It is believed that the mass-transfer results are applicable equally well to heat transfer under similar physical circumstances. Mass transfer is a component mechanism in liquid-phase catalytic processes, in leaching and adsorption operations, and in exchange processes such as occur in resin columns. The experimental technique involved partial dissolution of spherical and flake-shaped particles of difficultly soluble 2-naphthol in a rising stream of water. A precise analytical technique was developed for measuring the small concentrations of 2-naphthol in the exit stream involving dye formation and colorimetric analysis. The measured variables were water-flow rate, exit concentration, water temperature, particle and bed characteristics, and pressure drop. From these data correlating variables, including $J$ factor, Reynolds number, friction factor, and drag coefficient were interrelated. The range of experimental and correlative variables is: particle shape, modified spheres and flakes; particle size, $^1/_4$, $^3/_{16}$, and $^1/_8$-in. modified spheres, 8- to 10- and 14- to 18-mesh flakes (U. S. standard sieves) and mixed sizes; Reynolds number, 14 to 1755 in fixed beds; degree of expansion in fluidized beds, 36% voids in consolidated state to infinite expansion as represented by a single suspended particle; Schmidt group, 1200 to 1500; bed diameter, 4 in.; and consolidated bed depth, 5 to 24 in. The correlated results concerning mass transfer and friction in fixed and fluidized beds are discussed and compared. The importance of fraction void and the effect of nonuniformity of flow pattern as unifying relationships for mass transfer in fixed and fluidized beds and for single particles are discussed. Mass transfer per unit pressure drop in fixed and fluidized beds is also considered.

Authors' summary

1582. **J. Kaye, J. H. Keenan, W. H. McAdams, Report of progress on measurements of friction coefficients, recovery factors, and heat transfer coefficients for supersonic flow of air in a pipe,** Heat Transf. fluid Mech. Inst. 147–164 (May 1949).

Investigations were conducted in plastic tubes of very low roughness for a range of Mach numbers from 1.2 to 2.5 and for inlet Reynolds numbers on the order of $10^5$. The values of the mean apparent friction coefficient as obtained from the observed pressure gradient are somewhat smaller by one half or more than the corresponding incompressible-flow values. General agreement is observed with previously reported supersonic flow results. The recovery factor variation in the laminar boundary layer of Pohlhausen, namely $Pr^{1/2}$, is confirmed. For turbulent boundary layer, a variation from 0.79 to 0.865 for the Mach number range is reported. Data on the Stanton number are in accord with previous results. The authors report primarily their experimental data with very little attempt toward interpretation.

Newman A. Hall, USA

1533. **William M. Kays, Loss coefficients for abrupt changes in flow cross section with low Reynolds number flow in multiple tube systems,** Heat Transf. fluid Mech. Inst. 99–112 (May 1949).

The loss coefficients $K$ for incompressible fluids or low-Mach-number flows are given by the equation: $\Delta p/\gamma = KV^2/2g$. The expansion-loss coefficient $K_e$ is evaluated by an extension of the Borda-Carnot analysis, assuming a uniform velocity distribution at the downstream section $A_4$, taking the velocity distribution characteristic of long tubes and the Reynolds number for the upstream section $A_3$. For the evaluation of the contraction-loss coefficient $K_c$ the loss of mechanical energy is supposed to take place during the re-expansion only, inside the smaller tube; the contraction ratio is a function of $\sigma = A_3/A_4$.

The coefficients $K_e$ and $K_c$ are calculated for circular tubes and two-dimensional flow between parallel planes; square and triangular tubes are treated approximately. As the coefficients are defined on the basis of the bulk average velocity, negative values of $K_e$ are obtained for some cases of high $\sigma$. Experiments have been performed, using air as flow medium, in order to verify the applicability of the calculated values to heat-exchanger surfaces; the results confirm sustantially these values.

Giulio de Marchi, Italy

1534. **M. S. Brinn, S. J. Friedman, F. A. Gluckert, and R. L. Pigford, Heat transfer to granular materials,** Nucl. En. Power Aircr. Heat Transf. Lect. 1, 192–229 (Dec. 1948).

In many cases of cooling or heating of granular solids it is advantageous to keep the solids physically separated from the cooling or heating medium. One method of accomplishing the desired heat exchange is to allow the solid to flow in a settled condition through a single vertical pipe or a bundle of vertical pipes. Since there are no data available in the literature which can be used to predict the heat-transfer coefficients that might be obtained when using this method to heat or cool granular solids, the authors examined the factors which influence heat-transfer rates theoretically and experimentally (for several sorts of Ottawa and crushed Ilmenite sands). It was found that existing heat-transfer equations for rodlike flow of fluids correlated experimental data for the case of constant tube-wall temperature if the thermal conductivity of a stationary bed of the granular solid was used to evaluate the Graetz modulus. Visual evidence of rodlike flow was found.

Assuming rodlike flow, equations were developed and solved for the general case of a nonuniform tube-wall temperature caused either by significant resistance to heat flow in the heating or cooling medium, or by temperature changes in the jacket fluid accompanying countercurrent or parallel flow, or by combinations of these causes. Design charts have been prepared on the basis of these solutions and examples of their use presented.

Z. Horák, Czechoslovakia

**1585. B. H. Schultz, On the application of Reynolds' analogy and the heat-exchange factor to the design of heat exchangers: I, II, III,** Appl. sci. Res. Sec. A A1, no. 5–6, 387–416 (1949).

I. Reynolds' analogy between heat transfer and friction leads to the well-known equation (1)...$h = cfc_p\rho v$ ($f$ is friction). According to improved theories, for turbulent flow of gases in long straight ducts, the factor $c = 0.5$ is to be replaced by $c = 0.55$. It is shown that for laminar flow in long ducts, $c$ is equal to 0.3 for a circular, and to 0.4 for a flat rectangular cross section. Further, in very short ducts an approximate theory is shown to give values that are of the same order of magnitude. Though the equation (1) has an approximative character, it has the advantage of being simple and generally valid for gases flowing in ducts.

II. For heat exchangers in which the temperature differences between gas and wall decrease considerably, it is useful to introduce a "heat-exchange factor" $K$. A simple relation between $K$ and the pressure drop can be derived from the modified form of Reynolds' analogy given in part I. This relation is applied to some problems of optimum dimensions for heat exchangers.

III. The value of $c$ in the relation between $K$ and the pressure drop for turbulent flow is taken from experimental data given in the literature. The author's measurements in the transition region show that, though both heat exchange and friction may vary considerably with changing flow conditions at (or before) the entrance of the ducts, the factor $c$ does not vary appreciably. Recommended values of $c$ for flow of gases in ducts without special turbulence promotors are given in a diagram. In those cases where the curve for $f$ shows a pronounced dip, this is also found in the curve for the heat-exchange factor.

From author's summary by D. Jacovleff, Belgium

## Acoustics

**1586. Ernest Glen Wever, Theory of hearing,** New York: John Wiley and Sons; London: Chapman and Hall, 1949, xiv + 484 pp. Cloth, 5 × 7.25 in., $6.

Much of this book, dealing with the psychology of hearing, is of basic acoustic interest. Parts I and II (188 pp.) treat exhaustively the classical (resonance, place, and frequency) theories in their older and modernized forms. Part III (254 pp.) presents the "volley" theory, an integrated combination of the classical and the author's own ideas. (The volley principle assumes that high rates of discharge are a result of a combined action of many fibers working in concert.)

Ed.

**1587. Osman K. Mawardi, Measurement of acoustic impedance,** J. acoust. Soc. Amer. 21, 84–91 (Mar. 1949).

In this combined analytic and experimental paper, details are given on a simple means of comparing known and unknown acoustic impedances. An essentially constant-volume-velocity annular source feeds one plane end of a hard-walled cylindrical cavity (the known accoustic-impedance element) of dimensions small compared to a wave length. With the other plane end of the cavity closed by (1) a rigid wall, and then (2) the unknown acoustic impedance, the two corresponding voltage outputs $E'$ and $E''$ of a fixed probe microphone are compared in amplitude and phase. If the source and microphone present very high impedances to the cavity, and if the annular position of the source is properly chosen, then the unknown impedance $Z_m$ is expressed in terms of the cavity impedance $Z_1$ and microphone voltage by

$$Z_m = Z_1\,[1/((E'/E'') - 1)].$$

The author discusses the effects of departures from the idealizing assumptions, including effects of microphone-probe length and of cooling at the cavity walls. The experimental results compare very favorably with those from a more elaborate and absolute method. While primarily intended for acoustic materials, it appears easy to modify the equipment for measuring horns, tubes, the ear, etc.

Vincent Salmon, USA

**1588. Leonard Liebermann, Sound propagation in chemically active media,** Phys. Rev. 76, 1520–1524 (Nov. 1949).

A fluid containing chemically active ingredients in equilibrium may exhibit unique physical properties: the compressibility may depend on the rate of compression; similarly the specific heat may depend on the rate of temperature change. This rate dependence arises from the finite reaction rate of the chemical components. One phenomenon resulting from this behavior is the anomalous absorption and dispersion of sound. Expressions are given relating the chemical reaction rate, the equilibrium constant, and other chemical parameters, to the acoustical properties of the fluid. As an illustration of these relationships the unusual sound absorption in aqueous $MgSO_4$ solution is discussed. Calculation shows that the partial compressibility associated with the ionization process is of the correct magnitude to explain the observed absorption.

Author's summary

**1589. Thomas F. Protzman, Transition temperatures of methacrylate polymers at ultrasonic frequencies,** J. appl. Phys. 20, 627–730 (June 1949).

The velocity of sound in methyl methacrylate polymers (plexiglas) has been measured at temperatures from 24 to 90 C, and frequencies from three to eleven megacycles. Optical diffraction methods were used to determine the sound velocity within a maximum error of 1%. This low experimental error made possible a detailed study of the sound-velocity-temperature relationships. A previously unreported transition of a new type has been found. The sound velocity decreases linearly with increasing temperature, with an abrupt increase of slope occurring when a transition temperature is reached. The transition temperature is found to decrease linearly with frequency, falling from 64 C at three megacycles to 49 C at eleven megacycles. The observed transitions apparently are not manifestations of elastic relaxational effects, and are tentatively attributed to the excitation of some mode of molecular motion, for example, molecular vibrations.

Author's summary

**1590. R. B. Dingle, The theory of the propagation of second sound in helium II,** Proc. phys. Soc. Lond. 61, part 1, no. 343, 9–21 (July 1948).

The experiments of Peshkov (1946) in Russia, and Lane and his co-workers in America (1947), have demonstrated the existence of a new type of sound wave in liquid helium II for which temperature fluctuations rather than pressure fluctuations are the important phenomena. The essential assumptions of the theories developed by Tisza (1938, 1940, 1947) and Landau (1941, 1944) are: Helium II may be divided into two parts: a "normal" fluid which obeys the ordinary hydrodynamical equations with a coefficient of viscosity; and a superfluid for which the entropy is very small; it may be zero. As a consequence the superfluid

obeys hydrodynamical equations for which the curl of the velocity is everywhere zero so that the motion of the superfluid is always irrotational.

In the present paper the following problems are considered. The difference between an ordinary heat wave and a second-sound wave is shown to be that the former is dispersive whereas the second is not. It is shown that the kinetic and potential energy are equal for a second-sound wave. The attenuation due to heat-conduction processes is investigated and it is shown that the attenuation is high at 0 $K$ and at the $\lambda$ point 2.19 $K$, and has a minimum in-between. The reflection of second sound at a solid wall having a finite heat capacity and at a liquid-gas surface is investigated, and it is shown that standing waves can be set up between such boundaries. The effect of a finite-diameter tube on the velocity is shown to be small for second sound. The shape of the velocity temperature curve for second sound is discussed and is shown to depend on two parameters. These have not been accurately determined, and by selecting reasonable values it is possible to have a maximum between the $\lambda$ point and the maximum at absolute zero. Several criticisms are given of Tisza's method for deriving the velocity-temperature curve which predicts a zero velocity at 0 $K$. Warren P. Mason, USA

## Ballistics, Detonics (Explosions)

(See also Rev. 1529)

1591. **R. Zurmühl, The ballistics of the V2** (in German), Naturforschung und Medizin in Deutschland 1939–1946, vol. 7, 177–186. Dieterich'sche Verlagsbuchhandlung, Wiesbaden, 1948. Price DM10 ($2.40).

A brief account of German work on the ballistics of the V-2 rocket. The great majority of the papers to which reference is made are unpublished, and in most cases unavailable reports issued by the Technische Hochschule Darmstadt and the Heeren-anstalt Peenemünde.

*Courtesy of Mathematical Reviews* R. A. Rankin, England

## Soil Mechanics, Seepage

(See also Revs. 1452, 1472, 1516)

1592. **José A. Jimenez Salas, Soil pressures computation. A modification of the Newmark's method,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. 7, 30–34 (1948).

A table based upon the Boussinesq solution for a force on a semi-infinite body gives the vertical pressure at depth $z$, and on the axis of symmetry, due to uniform load of unit intensity on the annular area between concentric circles. With the table it is sufficient to superimpose a sketch of the area loaded on a chart consisting of a series of concentric circles, measure the percentage of each annular area covered by the sketch, and sum the products of percentage covered, load intensity, and tabular value for each annular area for the depth desired. Though similar to Newmark's method it has the advantage that pressures at any number of depths may be obtained with only one sketch and one chart.

Gerald Pickett, USA

1593. **József Jáky, Minimum value of earth pressure,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. 3, 61–65 (June 1948).

The author extends the classical theory of earth pressure to include new theorems. In particular he indicates that Coulomb's assumption can be proved to be correct and therefore ceases to be an assumption. He reduces Rebhann's theorem from the equilibrium of an elementary wedge.

The author gives formulas not only for the earth pressure on a retaining wall but also shows how to find the position and direction of the resultant force. He gives both analytical and graphical procedures for obtaining numerical values.

The cos $\phi$ which appears in Equation 9 and in the line immediately preceding should be replaced by sin $\phi$.

Gerald Pickett, USA

1594. **D. P. Krynine, Shearing stresses in supported and unsupported vertical slopes,** Proc. Highw. Res. Bd. Ann. Meet. 28, 443–449 (1948).

In a paper presented at the Second International Conference on Soil Mechanics and Foundation Engineering, the author gave a general view of the distribution of shearing stresses in a semi-infinite earth mass cut by a vertical slope. The present paper is a closer analysis of this problem. By means of elementary methods the author establishes relations between the horizontal shearing stresses and the lateral pressure within an earth mass bounded by a vertical slope, and explains the cracks, generally observed in the upper part of such a mass. The results obtained agree with the statements given by Terzaghi concerning such cracks.

Aurel A. Beleş, Rumania

1595. **Ch. Schraerer, W. Schaad, and R. Haefeli, Contribution to the shearing theory,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. 5, 12–19 (1948).

Methods of performing shear tests on saturated soils, and an approach to interpretation of results therefrom are treated in this paper. One type of test consists of shearing the soil specimen under conditions that permit drainage to occur and thus prevent the development of pore pressures. The results of this test are shown on a plot of shearing resistance versus normal load as a straight line through the origin and having an angle of inclination to the horizontal, $\phi_s$, defined as the angle of apparent internal friction. Another type of test is conducted by first consolidating the sample under a uniform pressure, $\sigma_v$, allowing the sample to rebound under a lesser pressure, $\sigma_1$, then shearing at a constant water content. The failure envelope for this test is a straight line having an intercept, $c$, on the vertical axis and is inclined to the horizontal at an angle $\phi_r$, defined as the angle of real internal friction. The failure envelope for the test at constant water content intersects the envelope for the drained system test at a point $T$, which is related to the preconsolidation pressure $\sigma_v$. If the test at constant water content is carried out at a normal pressure greater than $\sigma_v$, the failure envelope becomes a horizontal line which passes through point $T$. The failure envelopes for the tests at constant water content are a function of water content, and thus their locations will vary depending upon the preconsolidating pressure.

The test procedures described are not substantially different from those usually used in laboratory investigations of the strength of soils; both triaxial and ring shear tests are utilized. The intent of the paper is primarily to describe the method of interpretation of test results in an effort to obtain a better understanding of the shearing phenomenon in saturated soils.

Woodland G. Shockley, USA

1596. **R. R. Proctor, Construction and operational details for a simple machine to test soils in double shear,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. 7, 61–64 (June 1948).

Construction and operation of a double-shear machine, intended for compacted or undisturbed specimens of 4 in. or slightly less in diameter, are described and some typical results are given. It is pointed out that reliable shear strength data could save up to 50% of construction costs of compacted earth dams.

Hans F. Winterkorn, USA

**1597. A. Casagrande and W. L. Shannon, Research on stress-deformation and strength characteristics of soils and soft rocks under transient loading,** Harvard Univ. Publ. grad. School Engng. no. 447, 1-132 (June 1948).

This is a report on an experimental study of the effect of time of loading on the stress-deformation characteristics of four different clays, a sand, and Cucaracha rock from the Panama Canal Zone. The report contains a detailed description of the apparatus used and methods of testing. For the clays tested, the ratio of the compressive strength in transient loading (time $t_T = 0.02$ sec) to static loading ($t_s = 10$ min) varied from 1.4 to 2.6. For Cucaracha rock it was approximately 1.8, and for sand the scatter was so great that the results had little significance. The corresponding ratio for $t_T = 0.25$ sec and $t_s = 4$ hr for clay varied from 1.4 to 3.2, and for Cucaracha rock was constant at 1.6. No conclusive results can be drawn from these tests, but the authors state that on the average the transient compressive strength for clays and Cucaracha rock appear to be approximately twice that for static loading. For sands there seems to be little difference in the two strengths.

Eben Vey, USA

**1598. Liang-Sheng Chen, An investigation of stress-strain and strength characteristics of cohesionless soils by triaxial compression tests,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. 5, 35-43 (1948).

This paper describes the results of triaxial shear tests on dry, cohesionless soils covering a range of grain sizes from fine sand to gravel and having different amounts of angularity. The tests were performed at constant lateral pressures using a vacuum-type apparatus which permitted direct measurement of lateral strains on the specimens. Of particular interest are the methods used to prepare uniform test specimens at different degrees of density. Materials were placed using a specially constructed spoon, and compacted by varying blows of small tampers. Although it was assumed that the symmetric deformation of the specimens under load indicated uniform densities, the results were not checked with incremental density determinations.

Results of the study indicated that the angle of internal friction increased with increases in density, angularity of the grains, and uniformity coefficients for the soils tested. The compressibility of the soils increased with increasing angularity of grains but was not appreciably affected by grain size. Lateral strains of specimens under the test loads increased at a faster rate than corresponding axial strains.

Woodland G. Shockley, USA

**1599. A. W. Skempton, A possible relationship between true cohesion and the mineralogy of clays,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. 7, 45-46 (1948).

The author applies his formula, correlating the quotient of shear strength and consolidation pressure with what he calls "true cohesion" and "true angle of internal friction," to seven clay materials and finds that they fall into two groups, normal and inactive. The approach employed is too formalistic and not sufficiently fundamental, from the point of view of the physical and physicochemical mechanisms involved in shear, to justify the title and its implications.

Hans F. Winterkorn, USA

**1600. H. Raedschelders, Improvement of the method of calculation of the equilibrium along sliding circles,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. I, 68-72 (1948).

This analysis deals with slope failures along well-defined circular arcs when neither the cohesion nor the angle of internal friction are equal to zero, and when seepage forces or neutral pressures must be considered. The author describes two methods: that of calculating the moments about the center of rotation, and the method of slices. The first he rejects as incomplete since no study is made as regards the sum of horizontal or vertical forces, and the second he proves to contain some uncertain elements besides being tedious.

His solution is to draw the force polygon so that it closes, and having the forces pass by a common point to eliminate the moments of rotation. The new element in this work is that in the force polygon the required cohesion is such that the $\phi$-circle compatible with the conditions will be smaller than the one traced with the angle of internal friction of the material. The factor of safety can thus be described as both the ratio of the allowable cohesion to the developed cohesion, and the ratio of the allowable angle of internal friction to the one developed. The method is simple and fairly rapid.

Robert Quintal, Canada

**1601. Ralph E. Fadum, Influence values for estimating stresses in elastic foundations,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. 3, 77-84 (June 1948).

This paper deals with the problem of estimating the vertical normal stresses in quasi-elastic foundations due to building loads. Contributions to this subject from the theory of elasticity are briefly reviewed. Influence values derived from the Boussinesq and Westergaard solutions are presented for loading conditions that are frequently encountered in building settlement investigations, namely, for the case of loads applied at a point, distributed uniformly along a line of finite length, and distributed uniformly over rectangular and circular areas.

Influence values for some of the above loading conditions as computed from the Boussinesq solution have been previously published. To the writer's knowledge, influence values corresponding to the Westergaard solution have not appeared in the technical literature. A major purpose of this paper is to summarize for ready reference and in a form, which the writer has found convenient, influence-value data obtained from both the Westergaard and Boussinesq solutions that one may find desirable to have available when making a stress analysis for the computation of building settlements.

Author's summary

**1602. F. N. Hveem and R. M. Carmany, The factors underlying the rational design of pavements,** Highw. Res. Bd. Proc. 28, 101-136 (1948).

A discussion of the pavement-design problem is presented in four parts: (1-A) Analysis of the pavement-design problem; (1-B) Behavior patterns developed in masses of granular materials under load; (2) Mathematical relationships; (3) Testing of soils and bituminous mixtures; (4) Design procedure.

A chart is presented illustrating the process of analysis by means of which relationships are established between all of the major and minor factors involved. The history of soil technology is traced briefly and comparisons made between the attempts to reach a solution by mathematical analysis and the empirical approach employing the experimental method. It is shown that traffic-load effects are most nearly comparable to a strip loading, and the most probable planes of slip are calculated for soils possessing different proportions of friction and cohesion. Another part covers test procedures for evaluating and obtaining quantitative measurements of the essential properties of soils, photographs and sketches of test equipment, and methods of conducting tests. Procedures are given for designing pavement base and surface courses; typical examples are presented.

Rollie G. Fehrman, USA

**1603. Norman W. McLeod, A Canadian investigation of load testing applied to pavement design,** Amer. Soc. Test. Mat. Spec. tech. Publ. no. 79, 83-127 (June 1947, publ. 1948).

The methods of determining the thickness of flexible pavements for airport runways, as currently used, are considered conservative by Canada's Department of Transport which arranged for independent investigations. The paper deals with their findings which throw new light on pavement design. Repetitive plate-bearing tests were made on the subgrades, base courses, and flexible surfaces of the runways at ten Canadian airports, and useful correlations obtained. The results of these correlations indicate that limited load-test data may be extrapolated to other bearing-plate sizes between 12 and 42 in. in diam and probably beyond, and also to deflections up to 0.7 in. The results also indicated that bituminous surfaces have a load-carrying capacity approximately one-and-a-half times that of granular bases. The thickness design curves given in the paper indicate the required thickness of granular base for runways, taxiways, aprons, and turnarounds for a wide range of airplane wheel loads. These sets of curves are based on plate-bearing tests, and on cone-bearing, Housel-penetrometer, field California bearing-ratio, and triaxial compression tests.

S. K. Ghaswala, India

1604. **L. A. Palmer, Field loading tests for the evaluation of the wheel-load capacities of airport pavements,** Amer. Soc. Test. Mat. Spec. tech. Publ. no. 79, 9–40 (June 1947).

The author discusses the results of loading tests on plates of 15, 20, and 30-in-diam. The tests were continued for such a time that the total deflection was 0.2 in., or until the load was 75,000 lb. Typical data and relationships are given. The article is followed by an extensive discussion.

Later publications about this subject are: L. A. Palmer, The evaluation of wheel-load-bearing capacities progress report, Proc. Highw. Res. Bd. 1947, and L. A. Palmer and J. B. Thompson, Pavement evaluation by loading tests at Naval and Marine Corps air stations, Proc. Sec. Int. Conf. Soil Mech. 2, 222–236 (1948).

F. C. de Nie, Holland

1605. **Elwyn E. Seelye, W. D. Bailey, and S. D. Teetor, Methods of testing soils for runways and foundations,** Amer. Soc. Test. Mat. Spec. tech. Publ. no. 79, 41–50 (June 1947).

Rolling-load, plate-load, and California bearing-ratio tests are discussed, and results are given. The principal conclusion is that the rolling-load test and the California bearing-ratio are in fair agreement.

F. C. de Nie, Holland

## Micromeritics

1606. **W. Barth, Experiments to determine the size and the resistance of floating dust bodies** (in German), Ingen.-Arch. 16, no. 3-4, 147–152 (1948).

A method and apparatus are described for measuring the particle size of uniform powders in the range of sizes from about 5 to 320 microns. Air containing known concentrations of suspended powders is passed through a capillary tube, and the pressure drop is measured and compared with a similar flow of clean air. Particle diameters are computed from the data and appropriate equations for pressure gradient resulting from particle acceleration and friction. Directly measured particle sizes were compared with those measured by the method. Displaced and parallel results were obtained, indicating the need for calibration.

Richard H. Wilhelm, USA

1607. **Henry Stommel, Trajectories of small bodies sinking slowly through convection cells,** J. Mar. Res. 8, no. 1, 24–29 (May 1949).

The effect of a convective cellular motion in lakes on the settling of small particles is investigated theoretically for a mathematically simple case. In the absence of turbulent diffusion, and provided that the maximum upward velocity exceeds the settling velocity, particles in a region of predominantly upward flow are retained indefinitely, while all others eventually settle to the bottom. The effect of turbulence would be gradual transport of particles across the boundary out of the region of retention.

John S. McNown, USA

## Geophysics, Meteorology, Oceanography

**(See also Revs. 1557, 1607)**

1608. **Franz Baur, Introduction to global meteorology (Einführung in die Grosswetterkunde),** Wiesbaden, Dieterich'sche Verlagsbuchhandlung, 1948, viii + 168 pp. + 1 chart. Paper, 6 × 9 in., approx. $3.

This is a semipopular but convenient summary of the author's papers on the subject.

Besides the day-to-day weather changes, there are extended trends. A 5, 10, or 30-day period in any given region may be moister or drier, warmer or colder than average. Such anomalies are correlated to definite broad-scale flow patterns in the middle and upper troposphere. Conditions over large portions of the northern hemisphere determine the future weather in a particular location, mainly central Europe for the author's purposes.

The approach to the forecast problem is largely statistical. Three forecast intervals are chosen: 3–5 days, 10 days, and 30 days or more. Forecasts 3–5 days ahead are started by preparation of prognostic charts with the synoptic methods used in 24-hr forecasts. The author then adds:

(1) Analogues: Past situations with a flow pattern similar to that prevailing on the forecast day are studied for the developments that happened later.

(2) Multiple correlations based on the past record: the future pressure at a number of points is calculated with use of 4–6 variables, mostly pressure and temperature changes and anomalies preceding the forecast day.

(3) Harmonic analysis of the pressure at key stations and extrapolation of some of the components thought to be persistent. The same is done for the mean pressure integrated over larger areas.

In case of conflicting indications, the four approaches are reconciled qualitatively by the forecaster who bases the decision on his field experience.

The author's discussion on the prognoses for 10 days and more is much more vague. One gains the impression that the same methods are used, except that the prognostic charts prepared with the synoptic technique drop out and that "singularites" are included. For instance, rainfall tends to be low in northern Germany between May 28 and June 1.

The author, whose work is frequently overlooked in textbooks in the English language, deserves much credit for introducing consideration of "broad-scale weather" into forecast routines at an early date (1930's). It is clear, however, that he failed to develop forecast techniques that fully utilize the hemispheric weather connections noted. Such techniques are gradually being developed in the United States at this time.

Herbert Riehl, USA

1609. **Lester Machta, Dynamic characteristics of a tilted-trough model,** J. Met. 6, 261–265 (Aug. 1949).

A new wave solution of the vorticity solution for plane nondivergent horizontal motion in the atmosphere is presented. The treatment differs from that given previously by Rossby in that the

troughs and ridges do not lie along meridians but may possess a tilt. Owing to this feature the resulting waves produce a meridional transport of momentum and energy. Adaptation to the spherical form of the earth is not attempted. Ed.

**1610. Sidney T. Harding, Evaporation from free water surfaces,** in *Hydrology*, ed. by O. E. Meinzer, Dover Publications, 56–82 (1949).

This article presents a brief summary of mainly experimental results on evaporation problems, and a collection of theoretical and empirical formulas for the use of hydraulic engineers and meteorologists. The literature cited is restricted to American papers and a few theoretical British publications.

Horst Merbt, Germany

**1611. J. D. Cochrane and R. S. Arthur, Reflection of tsunamis,** J. Mar. Res. 7, no. 3, 239–251 (Nov. 1948).

It has generally been found that for appreciable reflection of water waves a large change in speed of propagation must occur over a distance which is small compared to a wave length. Since tsunamis are shallow water waves, they are propagated at the speed $\sqrt{(gh)}$. At the continental slopes and walls of deep oceanic trenches, reflection is to be expected, and the data examined confirm it well. In one case the observed wave heights were found to be in good agreement with those calculated from Lamb's and Jeffrey's theory, which also indicate that the reflected waves will, in general, be considerably smaller than the direct waves. This conclusion is in agreement with observations. Ed.

**1612. Ana Ioan, Mohorovicic's S-waves calculated by the Wiechert method** (in French), Bull. Sec. Sci. Acad. Roum. 30, no. 5, 270–274 (1947).

## Lubrication; Bearings; Wear

**(See also Rev. 1487)**

**1613. Mayo D. Hersey, Basic principles of lubrication, part 1: Fundamentals, part 2: Engineering applications, Diesel lubricating oils and basic principles of lubrication,** Trans. Amer. Soc. mech. Engrs. 71, 19–42 (Feb. 1949).

These two papers give a very excellent review of the basic principles of lubrication. The papers are not intended to be complete engineering discussions of all types of engineering and application problems. However, for the person who wishes a brief and concise description of the principles, these papers are highly recommended. Reference is made to many practical problems, and the bibliography is quite complete.

Erle I. Shobert II, USA

## Marine Engineering Problems

**(See also Revs. 1442, 1471, 1492)**

**1614. R. Brard, Introduction to the theoretical study of pitching in motion** (in French), Bull. Ass. tech. Marit. Aéro. 47, 455–479 (1948).

The purpose of the paper is to develop a more rigorous theory of the pitching motion of ships based on hydrodynamic considerations. The importance of the ratio $\gamma$ of the speed of advance $V$ to wave velocity $c$ for the flow pattern generated by the ship is emphasized.

The author derives first the two-dimensional complex velocity potential of a doublet, the speed of advance and the strength of which are arbitrary functions of time. From this complex potential, the potential $\varphi$ of a three-dimensional source is deduced by some appropriate artifices. This potential is simplified for a constant speed of advance and steady harmonic oscillations of the source strength, and evaluated by the method of stationary phases. The wave systems due to this pulsating source are thoroughly discussed; the wave pattern is essentially different for $\gamma \lesseqgtr {}^1/_4$. It is indicated how the different equations of a pitching and heaving ship can be derived from a system of oscillating sources.

Emphasis is put on an experimental curve which is presented with the intention to prove a decisive influence of the parameter $\gamma = V/c$ on the apparent masses. In the reviewer's opinion, the curve does not prove this point since it is derived under the fallacious assumption that the maximum amplitudes occur always at synchronism, which does not hold because of the variability of the exciting moment. The reproduced formula represents a fundamental result in hydrodynamics. Otherwise, most results of the paper have been anticipated by Haskind [Prikl. Mat. Mekh. 10 (1946); Math. Revs. 11, p. 228 (1950)] whose work goes much farther beyond the present one into applications. On page 456 it has been overlooked that Lagally's theorem ("la troisième méthode") cannot be applied to the study of ship oscillations since the motions are essentially not steady.

Georg P. Weinblum, USA

**1615. R. Sallé, Calculation of deformations and stresses in the keel angle of a seaplane** (in French), Bull. Assoc. tech. Marit. Aéro. 46, 569–585 (1947).

## Biomechanics

**(See Rev. 1586)**